

Problem I.2 ... pace yourself

3 points

On a rainy day, Matěj needs to get home as dry as possible. He has chosen the run-walk method, that is, he runs for half the time at speed $v_b = 15 \text{ km} \cdot \text{h}^{-1}$ and walks for half the time at speed $v_{ch} = 5 \text{ km} \cdot \text{h}^{-1}$, and he can choose when to take breaks from running. Small parts of the route pass through covered spaces (underpasses, arcades, etc.). Is it worthwhile for Matěj to plan his run-walk intervals so that he runs through or walks through these sections? The area of Matěj's effective horizontal cross-section is five times smaller than that of his vertical cross-section. Assume that the rain falls with a vertical speed $u_s = 20 \text{ km} \cdot \text{h}^{-1}$ and a horizontal speed $u_v = 5 \text{ km} \cdot \text{h}^{-1}$ in the direction of Matěj's motion. Neglect the relative motion of Matěj's limbs during motion.

Matěj got terribly wet which made him think about how to get less wet the next time.

Intuitively, we know that in a downpour it is usually advantageous to run in order to spend as little time in the rain as possible. In our case, however, the wind works in Matěj's favor because it blows in the direction of his motion and even has the same horizontal speed as his walking speed. Thus, while walking, he is hit by raindrops only from above and catches none on his front side, as he would while running.

We approximate Matěj by a rectangular cuboid with a frontal cross-sectional area $5S$ and a horizontal cross-sectional area S , neglect the relative motion of his limbs, and assume homogeneous rain. When moving at speed v , Matěj receives a rain flux Su_s from above, while his front (or back) surface catches a flux $5S|v - u_v|$. Thus, in total,

$$Q(v) = Su_s + 5S|v - u_v|.$$

Since Matěj needs to cover a given distance, the quantity determining which speed is more advantageous in the rain is not the absorbed rain flux, but the absorbed rain flux per unit distance traveled. Therefore, we must divide Q by Matěj's speed.

$$\frac{Q(v)}{v} = 4S\frac{u_v}{v} + 5S\left|1 - \frac{u_v}{v}\right|,$$

where we have directly used the relation $u_s = 4u_v$. It now suffices to substitute the speeds $v_{ch} = u_v$ and $v_b = 3u_v$ and determine in which case the result is lower. The resulting amounts of rain collected per meter are

$$\frac{Q(v_1)}{v_1} = 4S, \quad \frac{Q(v_2)}{v_2} = \frac{14}{3}S \doteq 4.67S.$$

We see that while walking, Matěj collects less rain per meter than while running. To minimize the total amount of rain collected during the run-walk method, it is advantageous to assign the

more costly phase (running) to the covered section, where it is not raining, and leave the less costly walking phase for the rainy part of the route.

Matěj Mezera
m.mezera@fykos.org

FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of CUNI MFF, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports of the Czech Republic.

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