

Problem I.1 ... feeling the air dome

3 points

In an air dome, there is an excess pressure of $\Delta p = 310$ Pa relative to the surroundings. The atmospheric pressure outside the dome is $p_{\text{out}} = 950$ hPa. If we were very sensitive, leaving the hall would give us a sensation similar to a sudden change in elevation. What altitude difference does this pressure difference correspond to? *Karel was thinking about air domes.*

Since this is a simple problem, we assume that the participant will find a suitable barometric equation,¹ and use it to solve the problem, commenting on its applicability. If a source is provided, there is no need to discuss the full derivation.

The excess pressure given in the problem statement is very small compared with the atmospheric pressure (by more than two orders of magnitude), so we may assume that the corresponding altitude difference will not be large either. For small altitude differences, the isothermal barometric equation can be used with sufficient accuracy. It has the form

$$p = p_0 e^{-hgM/(RT)},$$

where p_0 is the absolute pressure at the selected reference altitude (in our case, this is the pressure outside the dome, which we denote by p_{out}), g is the gravitational acceleration, M is the mean molar mass of air ($M \doteq 29.0 \text{ g}\cdot\text{mol}^{-1}$, which is closer to that of nitrogen than oxygen), $R \doteq 8.314 \text{ J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$ is the molar gas constant, and h is the altitude difference we seek, measured from the level of the dome. The higher the altitude, the lower the pressure.

We rearrange the equation for our case. We express the pressure inside the dome as $p = p_{\text{out}} + \Delta p$:

$$\begin{aligned} \frac{p_{\text{out}} + \Delta p}{p_{\text{out}}} &= e^{-hgM/(RT)}, \\ \ln\left(\frac{p_{\text{out}} + \Delta p}{p_{\text{out}}}\right) &= -\frac{gM}{RT}h, \\ h &= -\frac{RT}{gM} \ln\left(\frac{p_{\text{out}} + \Delta p}{p_{\text{out}}}\right) \doteq 26 \text{ m}. \end{aligned}$$

The temperature was not specified in the problem statement, so the numerical results may differ. In this solution, we estimate the temperature; for the numerical calculation, we may use, for example, $T = 275 \text{ K}$, since air domes are used mainly in winter and this temperature is appropriate. However, changing the temperature by several $^{\circ}\text{C}$ does not affect the order of magnitude of the result and only changes the subsequent significant digits.

The excess pressure inside the dome corresponds to a situation in which a person inside is 26 m lower. Thus, upon leaving the dome, we experience a sensation as if we had climbed 26 m higher. This is a negligible difference that can be perceived only by very sensitive individuals.

For comparison, in air traffic control, a rule of thumb is to estimate a pressure decrease of 1 hPa per 8 m. This approach gives a result of approximately 25 m, which is less accurate but very useful for quick mental calculations.

¹The barometric equation describes the decrease in atmospheric pressure with increasing altitude.

Alternative solution

Upon leaving the dome, the pressure decreases by $\Delta p = 310 \text{ Pa}$. Since this difference is very small compared with the atmospheric pressure, it is sufficient to use the approximate relation for small altitude differences $\Delta p \doteq \rho gh$. For the air density, we use the approximate value $\rho \doteq 1.2 \text{ kg}\cdot\text{m}^{-3}$. We obtain

$$h \doteq \frac{\Delta p}{\rho g} = \frac{310 \text{ Pa}}{1.2 \text{ kg}\cdot\text{m}^{-3} \cdot 9.81 \text{ m}\cdot\text{s}^{-2}} \doteq 26 \text{ m}.$$

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