

Problem VI.E ... falling object

12 points

Measure how distance traveled and velocity depend on time during a free fall. Use a light symmetric object (ideally a ping-pong ball) and drop it from a height of at least 5 meters so that the effect of air resistance becomes measurable. Compare the recorded dependencies with an appropriate theoretical model.

Hint: Record the fall on video and analyze it using suitable software, such as Tracker.

Jarda was imagining what would it be like to jump out of a dormitory window.

Theory

The aim of the task is to observe the effect of air resistance during the free fall of an object from the initial height H_0 . Therefore, the equations

$$\begin{aligned}s &= \frac{1}{2}gt^2, \\ v &= gt,\end{aligned}$$

which describe the dependence of the distance traveled s and the velocity v on the time from release t , and which apply for motion with constant acceleration g caused by the gravitational field at the place of the fall, are not sufficient. In our case, we must take into account not only the gravitational force F_g but also the effect of the drag force F_o , which acts against the direction of motion. The equation of motion given by Newton's second law is thus

$$ma = F_g - F_o = mg - F_o,$$

where m is the mass of the object and a its acceleration downwards. Let us base our theoretical model of motion on a drag force of the form

$$F_o = \frac{1}{2}CS\rho v^2,$$

where C is the drag coefficient, S is the cross-sectional area of the object, ρ is the density of the medium in which the object falls, and v is its instantaneous velocity. In our case, we drop a ping pong ball, which has the shape of a sphere with radius $r = 2.0$ cm. This corresponds to a drag coefficient $C \doteq 0.5$ and a cross-sectional area $S = \pi r^2 \doteq 0.00126$ m². The mass of the ping pong ball is $m = 2.7$ g. We chose this object for its well-defined properties, whose values are guaranteed by the manufacturer, and its symmetrical shape. We release it in air at temperature 15 °C, which corresponds to density $\rho = 1.23$ kg·m⁻³ according to Wikipedia.¹

The equation of motion for the ping pong ball is therefore

$$ma = mg - \frac{1}{2}CS\rho v^2 = mg - kv^2.$$

Its solution using differential calculus is for example given on the pages of the Physics Olympiad². The dependence of velocity on time from release is

$$v(t) = \sqrt{\frac{gm}{k}} \tanh\left(\sqrt{\frac{gk}{m}}t\right)$$

¹https://en.wikipedia.org/wiki/Density_of_air

²<http://fyzikalniolympiada.cz/texty/odpor.pdf>

and the distance traveled

$$s(t) = \frac{m}{k} \ln \left(\cosh \left(\sqrt{\frac{gk}{m}} t \right) \right).$$

As stated in the previous section, we chose a ping pong ball with mass $m \doteq 2.7$ g and radius $r \doteq 2.0$ cm as the falling object. We dropped it in air at temperature 15°C , which corresponds to a density $\rho \doteq 1.23$ kg·m⁻³.

We chose the railing of a balcony of a family house at height $H = 7$ m as the release point. The fall was repeated five times for statistical evaluation. The trajectory of the ball was recorded by a digital camera at 4K resolution with a frame rate of $f = 100$ fps, corresponding to the time between recorded positions $T = 0.01$ s. The camera was placed at a distance of 10 m from the base of the house on a tripod at a height of 140 cm. A measuring tape of length 2.00 m was also placed in the frame, serving as calibration for converting image coordinates to real distances.

The fall recordings were processed using the program *Tracker*. This program can be “trained” to recognize the object whose position changes between frames. The program automatically calculates the current position of the object in the given video frame. Using the measuring tape, calibration was introduced, so the resulting distance given by the program was converted to the actual distance the object traveled. The measured dependencies were exported to a text file.

For further processing, the programming language *Python* was used. Velocity as a function of time was determined using the function `numpy.gradient()`, which computes the derivative. This is very convenient since we did not measure velocity directly, only distance as a function of time. However, since velocity is the derivative of distance with respect to time, we can obtain it using this function.

The results could depend strongly on determining the frame at which the ball was released. At the start, the ball moves very slowly, so it remains approximately in the same position. In this case, the above-mentioned dependence of velocity on time helps us with data processing. Initially, the velocity is zero while the ball is held in the hand. Once it is falling, the velocity grows approximately linearly with time, because the drag force dependent on velocity is still small. The initial time of the fall can thus be approximated by a straight line symbolizing free fall without drag forces. The moment this line crosses zero velocity indicates the release of the ball. From that moment on, we will work with all data.

Results and discussion

Let us plot the dependence of distance on time from release for all trials (Figure 1) and compare it with the distance the ball would travel in vacuum (“Fall in vacuum”) and the distance according to our theoretical model (“Fall theory”). We see that the distance traveled is for any time less than in vacuum and that the ball is indeed slowed down. This delay grows the longer the ball falls, as the effect of the drag force becomes more pronounced.

For comparison with our model, we also plot the velocity dependence on time in Figure 2, where besides the model neglecting air resistance (“Fall in vacuum”), the function considering drag force (“Fall theory”) is again shown. We see that the velocity data points are noisier, which is due to the numerical differentiation that amplifies measurement noise. It is evident, however, that velocity does not grow linearly with time, as it would in vacuum, but is always lower and appears to saturate somewhat, i.e., converge to a constant value.

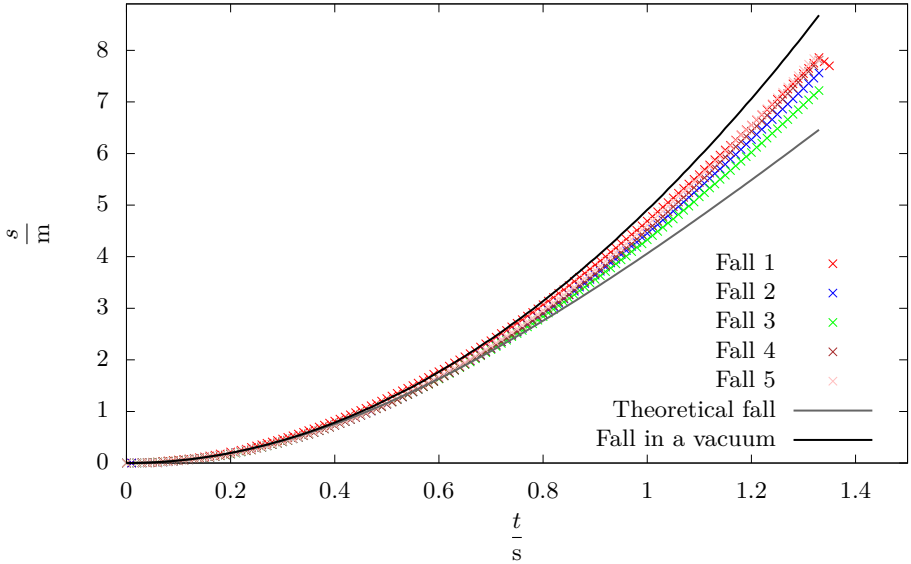


Figure 1: Dependence of traveled distance on time for different trials. The distance is always less for the ball falling through air than in free fall in vacuum.

On the other hand, the velocity function ("Fall theory") according to our theory and values given in the previous section predicts velocity lower than our measured values. We must therefore state that our data do not agree with the theoretical model. A similar effect is, of course, seen in the dependence of distance on time. There may be several explanations. Systematic errors arose because we did not correct for the effect of the three-dimensional space recorded on the camera chip. The real traveled distance near the release point is different from the point of impact, although it may appear the same on the recording. This is due to the angle under which we observed the trajectory of the fall. The measured data thus may not exactly reflect reality.

Furthermore, we might not have used the correct physical model. The use of drag force

$$F_o = \frac{1}{2}CS\rho v^2,$$

is limited to turbulent flow of fluid around the body. A condition for this flow regime is typically the *Reynolds number* $R = vr/\nu > 1\,000$, where ν is the kinematic viscosity of the fluid. This number is proportional to v , but v is initially zero and then small, so this regime does not occur at first. At least in the initial phase, the drag force is probably not proportional to the square of velocity, which leads to a deviation from the assumed function.

Wind could also have played a role, slightly displacing the ball during flight. Its direction need not have been strictly parallel to the ground but could have affected vertical motion as well.

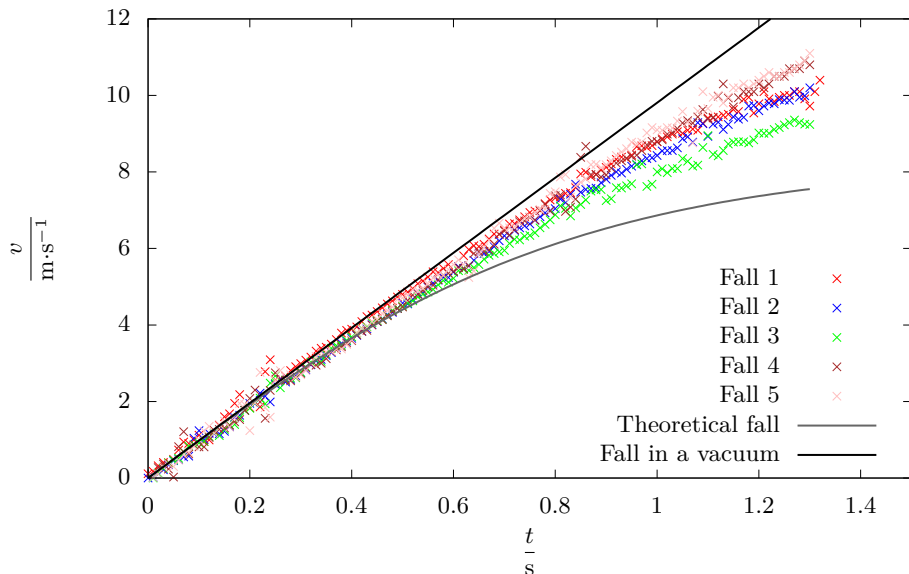


Figure 2: Velocity dependence on time for different trials. Velocity is always lower for the ball falling through air than in free fall in vacuum.

Even though our model does not quantitatively match the measured data, we see good qualitative agreement. At the start for short times t the velocity increases linearly with proportionality constant g , as seen by the agreement of experimental data with the "Fall in vacuum" function. This corresponds to negligible drag force. At longer times, approximately for $t > 0.5$ s, the velocity is lower than in vacuum. This difference increases with time.

Conclusion

We measured the free fall of a light symmetric object—a ping pong ball—in air. We compared the measured dependence of distance on time with free fall in vacuum, where no drag forces act, and the traveled distance was always shorter when falling in the air. For comparison with a theoretical model, which assumed a quadratic dependence of the drag force on velocity, we plotted velocity as a function of time.

Our data showed qualitative agreement with the theoretical model—at low velocities, velocity increased linearly with time, but acceleration gradually decreased, corresponding to drag force growing with velocity. Thus, we observed the effect of air resistance. However, quantitative agreement with the theoretical model was not found, which may be caused by an

inappropriate description or systematic measurement error.

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