Problem VI.3 ... pancake

5 points

Jarda decided to use centrifugal force to make a pancake. Onto a smooth, symmetric circular pan of radius R, rotating with angular velocity ω, he quickly poured batter of mass m onto the center. Initially, it formed a cylinder of height h_0 and radius r_0 ($h_0 \ll r_0 < R$). The surface tension between the batter and the surrounding atmosphere is σ_1, between the batter and the pan is σ_2, and between the pan and the atmosphere is negligible. Energy is supplied to maintain the rotation of the pan with constant power P. Assume that immediately after being poured onto the pan, the layer of batter rotates together with it, and gravity ensures only that the layer always has the shape of a cylinder; otherwise, the gravitational potential energy may be neglected. After how long from the start of rotation does the batter reach the edge of the rotating circle?

Jarda wants to eat only thinnest (and most symmetrical!) of pancakes.

The problem is motivated by an important technological process that is not related to gastronomy, but to the lithographic production of semiconductor chips. This is a process lasting several months and taking place in very clean spaces. First, a silicon wafer is covered with a layer of photosensitive material, the so-called photoresist. Through a photomask (a template with the circuit design), the pattern is then transferred onto the wafer using UV lasers. The wafer is then chemically treated, which removes the exposed (or unexposed) photoresist and creates the desired pattern. Finally, the material in unprotected places is removed (etching) or, conversely, added (deposition) using chemical or physical processes. This permanently creates the circuit structure.

In the first step, the photoresist is applied by spin coating. A very small amount of liquid photoresist is applied to the wafer, and the wafer is then spun at high speed (often even thousands of revolutions per minute). Centrifugal force spreads the liquid into a very thin and uniform layer over the entire surface. Depending on the particular technology, the layer thickness ranges from tens of nanometers to several micrometers.

After this short introduction, we can proceed to solving the problem. The kinetic energy of rotational motion is determined as

$$E_\mathrm{k} = \frac{1}{2} I \omega^2 \,,$$

where the moment of inertia of the cylinder is $I = (1/2)mr^2$, with r denoting its current radius. Initially, the pancake radius is r_0, and at the end (when it reaches the edge of the circle) it is R. The rotational energy of the batter, therefore, increases by

$$\Delta E_\mathrm{k} = \frac{1}{4} m R^2 \omega^2 - \frac{1}{4} m r_0^2 \omega^2 = \frac{1}{4} m \omega^2 (R^2 - r_0^2) \,.$$

The surface energy remains. We obtain it by multiplying the area by the corresponding surface tension, that is, $E_\sigma = S\sigma$. Since $h_0 \ll r_0 < R$, we can neglect the lateral surface area compared with the area of the bases. The area of one base is $S = \pi r^2$, so the total surface energy of the batter is $E_\sigma = \pi r^2 (\sigma_1 + \sigma_2)$. We obtain the change in surface energy between the final and initial states as

$$\Delta E_\sigma = \pi R^2 (\sigma_1 + \sigma_2) - \pi r_0^2 (\sigma_1 + \sigma_2) = \pi (\sigma_1 + \sigma_2)(R^2 - r_0^2) \,.$$

If the surface tension between the pan and the atmosphere were nonzero, we would have to subtract the change in this energy. However, the problem statement says that we should neglect this effect.

The total energy, therefore, increases by

$$\Delta E = \Delta E_\mathrm{k} + \Delta E_\sigma = \left(\frac{1}{4} m\omega^2 + \pi(\sigma_1 + \sigma_2)\right)\left(R^2 - r_0^2\right).$$

It is enough to divide this change by the power P, which gives the time t that the process takes:

$$t = \frac{\Delta E}{P} = \frac{1}{P}\left(\frac{1}{4} m\omega^2 + \pi(\sigma_1 + \sigma_2)\right)\left(R^2 - r_0^2\right).$$

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