Problem VI.2 ... perforated bilayer 3 points

Both plates of a capacitor are perforated with a small hole at the same height. The plates are then brought very close to each other and charged to a potential difference U, thereby forming a charge bilayer. An electron approaches the hole in the plates with velocity v, at an angle α with respect to the normal, and is closer to the positive plate. At what angle does it emerge from the hole on the other side? What would this angle be if, initially, it were directed toward the negative plate? *Jarda's bug screen has a hole in it.*

We will neglect edge effects near the holes in the plates. When the electron moves outside the capacitor, no electric force acts on it, because the contributions from the positive and negative plates cancel each other. The electron, therefore, moves along a straight line. Between the plates, however, an electric force acts on it; the electric field intensity points from the positive plate to the negative plate, so the force acting on the electron points toward the positive plate. The velocity parallel to the plates does not change.

Initially, the parallel component of the electron velocity is

$$v_{\parallel} = v \sin \alpha \,,$$

and the perpendicular component is

$$v_{\perp,i} = v \cos \alpha \,.$$

During the passage through the capacitor, only the perpendicular velocity component changes. The simplest approach is to use the law of conservation of energy. During the passage from the positive plate to the negative plate, the electric field does work

$$W = -eU \,,$$

so

$$\frac{1}{2} m v_{\perp,f}^2 - \frac{1}{2} m v_{\perp,i}^2 = -eU \,.$$

From this, we obtain

$$v_{\perp,f} = \sqrt{v^2 \cos^2 \alpha - \frac{2eU}{m}} \,.$$

For the electron to pass through to the other side at all, the condition

$$v^2 \cos^2 \alpha \geq \frac{2eU}{m} \,.$$

must hold. If this condition is not satisfied, the electron stops in the field and returns through the first hole. Since it has the same tangential velocity, it returns at an angle $-\alpha$ with respect to the normal. Thus, a mirror reflection occurs.

The angle β after the electron exits the capacitor is determined from the relation

$$\tan \beta = \frac{v_{\parallel}}{v_{\perp,f}} = \frac{v \sin \alpha}{\sqrt{v^2 \cos^2 \alpha - \frac{2eU}{m}}} \,.$$

If the electron were initially directed toward the negative plate, the electric force would instead accelerate it. In that case,

$$\frac{1}{2} m v_{\perp,f}^2 - \frac{1}{2} m v_{\perp,i}^2 = +eU$$

and therefore

$$v_{\perp,f} = \sqrt{v^2 \cos^2 \alpha + \frac{2eU}{m}}\,,$$

from which we get

$$\tan \beta = \frac{v_{\parallel}}{v_{\perp,f}} = \frac{v \sin \alpha}{\sqrt{v^2 \cos^2 \alpha + \frac{2eU}{m}}}\,.$$

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