Problem V.5 ... fridge magnet

8 points

A permanent bar magnet with dipole moment μ, mass m, radius r, and length l is attached horizontally to a refrigerator. What is the heaviest weight that can be hung from its end if the coefficient of friction between the magnet and the refrigerator is f? For simplicity, assume that the refrigerator forms a half-space of perfectly magnetizable metal and that the magnetic field of the magnet is dipolar and symmetric with respect to its body.
Hint: *Use a point dipole.*

Jarda collects fridge magnets.

In the material of the refrigerator, the orientation of individual magnetic domains changes so as to compensate for the external magnetic field produced by the bar magnet. This situation can be described using a fictitious magnet located somewhere inside the refrigerator, whose field corresponds to the field generated by the refrigerator's ferromagnetic material.

In electrostatics, we are familiar with analogous behaviour: an electric charge near a conducting planar surface induces a redistribution of charge whose field can be described by a fictitious charge of opposite sign located on the other side of the surface. This charge is called the image charge.

Magnetostatics shares many features with electrostatics. Although magnetic monopoles do not exist, unlike electric charges of a single sign, we may introduce them formally and compute fields and forces using them; mathematically, this is equivalent.

Instead of the magnetic dipole represented by the bar magnet in the problem, we therefore consider two monopoles, two magnetic quantities Φ (analogous to the electric charge q) of opposite sign separated by a small distance Δ, for which

$$|\mu| = \Phi\Delta\,.$$

We assume that these magnetic quantities obey laws analogous to electrostatics. If we therefore assume the existence of an image magnet behind the refrigerator wall, we may equivalently represent it by two magnetic quantities of opposite sign located behind the refrigerator, otherwise identical to the real magnet.

The task asks about the stability of a configuration of objects, so we must determine the acting forces and torques. We begin with the magnetic force by which the magnet is attracted to the refrigerator. The distance of the first magnetic charge of the real magnet from the refrigerator is $l/2 - \Delta/2$, while the second, of opposite sign, is at distance $l/2 + \Delta/2$. The first charge is attracted to the nearer image charge and repelled by the farther one; for the second real charge the situation is reversed. The total force between the real magnet and its image is therefore

$$F = \frac{\mu_0\mu_\mathrm{r}}{4\pi}\Phi^2\left(\frac{1}{(l-\Delta)^2} - \frac{1}{l^2} - \frac{1}{l^2} + \frac{1}{(l+\Delta)^2}\right)\,.$$

Recall that this calculation is identical to that for the electrostatic Coulomb interaction, differing only in the prefactor. Instead of $(1/\varepsilon_0)$ we have the permeability μ_0, μ_r, where μ_r is the relative permeability of the magnet material. The dipole behind the refrigerator wall is the image dipole, so the permeability of that region is the same as on the magnet's side. We also note that we compute the force between two magnets, hence we do not consider internal forces between the magnetic charges within each dipole.

We assume point dipoles, so the separation Δ between charges in each magnet is much smaller than l, i.e., $\Delta \ll l$. We rewrite the expression as

$$F = \frac{\mu_0 \mu_\mathrm{r}}{4\pi} \frac{\Phi^2}{l^2} \left(-2 + \frac{1}{\left(1 - \frac{\Delta}{l}\right)^2} + \frac{1}{\left(1 + \frac{\Delta}{l}\right)^2} \right) ,$$

and use the Taylor expansion $(1+y)^{-2} \approx 1 - 2y + 3y^2 - \ldots$, which simplifies the expression significantly. Substitution yields

$$F \approx \frac{\mu_0 \mu_\mathrm{r}}{4\pi} \frac{\Phi^2}{l^2} \left(-2 + \left(1 - 2\left(-\frac{\Delta}{l} \right) + 3\frac{\Delta^2}{l^2} \right) + \left(1 - 2\frac{\Delta}{l} + 3\frac{\Delta^2}{l^2} \right) \right) = \\ = \frac{\mu_0 \mu_r}{4\pi} \frac{6\Phi^2 \Delta^2}{l^4} \,.$$

We see that the Taylor expansion was indeed needed up to second order, since the first two orders cancel. Using the definition of the magnetic dipole moment $\mu = \Phi\Delta$, we obtain

$$F = \frac{\mu_0 \mu_\mathrm{r}}{4\pi} \frac{6\mu^2}{l^4} \,.$$

We have thus obtained the force with which the magnet is attracted perpendicularly toward the refrigerator.

To prevent the magnet from sliding down the refrigerator, the friction force $F_\mathrm{t} = fF$ must exceed the gravitational force of the magnet and the attached mass. In the limiting case of equality,

$$F_\mathrm{t} = fF = (m + M)\,g \quad \Rightarrow \quad M = \frac{fF}{g} - m \,,$$

and after substitution,

$$M = \frac{\mu_0 \mu_\mathrm{r}}{4\pi} \frac{6\mu^2 f}{g l^4} - m \,.$$

There is another possible failure mode in which the magnet with the attached mass detaches from the refrigerator: rotation about the lower edge. For this case we analyse torques. One contribution is the magnetic force with moment Fr, where r is the radius of the magnet. Opposing this are the gravitational torques of the magnet $mgl/2$ and the mass Mgl. The limiting mass M is found from torque balance:

$$Fr = \frac{mgl}{2} + Mgl \quad \Rightarrow \quad M = \frac{Fr}{gl} - \frac{m}{2} \,,$$

which after substitution gives

$$M = \frac{\mu_0 \mu_\mathrm{r}}{4\pi} \frac{6r\mu^2}{g l^5} - \frac{m}{2} \,.$$

The resulting value of M is the smaller of the two computed limits. This would of course depend on the specific numerical values in the problem.

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