Problem V.1 ... enchanted beans

3 points

Marek bought enchanted beans from a strange trader at the train station; once they grow, they are supposed to lead to a castle of magical giants somewhere high in the sky. How high can the giants be so that Marek can reach them, if all the carbon for the bean plant comes from atmospheric carbon dioxide? Assume that the stem is pure cellulose with density $\rho = 1.56\,\mathrm{g\cdot cm^{-3}}$ in the shape of a cylinder with base $R = 1.0\,\mathrm{km}$. Estimate the amount of carbon dioxide in the atmosphere and compare it with more accurate data.

Marek contemplated selling his only cow.

Let us attempt a rough estimate of the mass of carbon dioxide in the atmosphere. The total mass of the atmosphere is approximately $M_\mathrm{atm} \approx 5.2{\cdot}10^{18}\,\mathrm{kg}$; let us take its molar mass to be the standard value of $29\,\mathrm{g\cdot mol^{-1}}$. The molar mass of carbon dioxide is approximately $44\,\mathrm{g\cdot mol^{-1}}$, and the concentration of carbon dioxide in the atmosphere is given as 420 ppm (i.e., $4.2 \cdot 10^{-4}$ of the molecules), so the total mass is

$$M_{\mathrm{CO_2}} \approx M_\mathrm{atm} \cdot 4.2 \cdot 10^{-4} \cdot \frac{44}{29} = 3.3 \cdot 10^{15}\,\mathrm{kg}\,.$$

The pure carbon fraction is, from the molar masses, $M_\mathrm{C}/M_{\mathrm{CO_2}} \approx 12/44 = 3/11$, so $M_\mathrm{C} \approx 9.0 \cdot 10^{14}\,\mathrm{kg}$. Conversely, for cellulose with the formula $C_6H_{10}O_5$, the mass fraction is

$$M_\mathrm{C}/M_\mathrm{cellulose} \approx \frac{6 \cdot 12}{6 \cdot 12 + 10 \cdot 1 + 5 \cdot 16} \approx 0.44\,,$$

from which $M_\mathrm{cellulose} \approx 2.0 \cdot 10^{15}\,\mathrm{kg}$, and because we know the density from the problem statement to be $\rho = 1.56\,\mathrm{g\cdot cm^{-3}} = 1560\,\mathrm{kg\cdot m^{-3}}$, we obtain the total plant volume as $V \approx 1.3 \cdot 10^{12}\,\mathrm{m^3}$.

For the volume of a cylinder, we have

$$V = \pi R^2 h\,,$$
$$h = \frac{V}{\pi R^2} \approx 420\,\mathrm{km}\,,$$

where R is the base radius given in the problem statement.

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