Problem IV.S ... detectors of ionizing radiation 10 points

1. *In the attached file[1] you will find the measured lifetimes of atoms of the radioactive isotope ^{214}Po. The time is in nanoseconds. Determine the half-life of this isotope. – 3 points*

2. *An electron with kinetic energy $150\,\mathrm{keV}$ flew into the detector and transferred all of its energy to the detector. Determine the uncertainty of the measured energy if the detector was*

   - *a gas detector with argon;*
   - *a silicon semiconductor detector;*
   - *a scintillator $LaBr_3$(Ce).*

   Consider that the source of uncertainty is only the fluctuation in the number of free charge carriers or photons created. Also assume that the detector collects all charge carriers (or all photons in the case of a scintillator). For the scintillator, consider the Fano factor $F = 1$. – 2 points

3. *Consider the AMS-02 experiment aboard the International Space Station (ISS). In addition to other detectors, it includes a transition radiation detector, a RICH detector, and a magnetic spectrometer with 7 layers of silicon detectors. The RICH detector contains NaF with a refractive index of 1.33 and aerogel with a refractive index of 1.05.*

   In addition to measuring position with an accuracy of $10\,\mathrm{\mu m}$, the silicon detectors in the magnetic spectrometer can also measure energy loss per unit length $\mathrm{d}E/\mathrm{d}x$. Describe qualitatively how these three detectors can be used to distinguish between the following four particles: electron, proton, antiproton, and helium nucleus ^{4}He. Each particle has the same kinetic energy $3\,\mathrm{GeV}$. – 2 points

4. *From the Bethe-Bloch equation, derive the β factor at which the energy loss per unit length $\mathrm{d}E/\mathrm{d}x$ is the lowest (minimum ionizing particle). Neglect the correction terms $\delta(\beta)$ and $C(\beta)$ and also assume that the equation in this form also applies to electrons. Do not be afraid of numerical solutions. You can use the average excitation energy $I = 92.2\,\mathrm{eV}$, which we calculated for nitrogen using the equation from the series text. In this way, we can approximate the passage through air. What are the kinetic energies of a MIP electron, a MIP muon and a MIP proton? – 3 points*

A muon has just passed through Jindra.

Subproblem 1

The analysis could be carried out in several ways. It was possible to create a histogram of lifetimes and then fit an exponential $A\exp(-\lambda t)$ (see Figure 1). This can also be done in Excel or another spreadsheet program. From the decay constant $\lambda = 0.00381\,\mathrm{\mu s^{-1}}$, we can calculate the half-life

$$T_{1/2} = \frac{\ln 2}{\lambda} = 182\,\mathrm{\mu s}\,.$$

[1] https://drive.google.com/file/d/1V4F9egbkg8AmpQhoiTW303rTghtbb2v8/view

From the shape of the histogram, we can notice that the random-detection background is negligible, so we can also calculate the average lifetime, which under these conditions gives us a good estimate of the mean lifetime $T = 242.5\,\mu\mathrm{s}$. The half-life is $T_{1/2} = T\ln 2 = 168\,\mu\mathrm{s}$. For comparison, the tabulated half-life of ^{214}Po is $164.3\,\mu\mathrm{s}$.

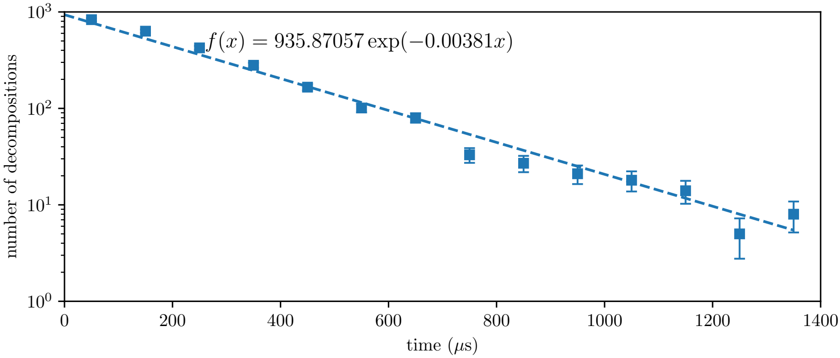

Figure 1: Histogram of the lifetimes of the isotope ^{214}Po with a fitted exponential. The y-axis is logarithmic, so in this representation the exponential appears as a straight line. The analysis was carried out in a spreadsheet program.

Subproblem 2

The total energy absorbed in the detector is $E = 150\,\mathrm{keV}$. In the case of the argon detector, from the table in the series text, we found the average ionization energy $\varepsilon = 26\,\mathrm{eV}$ and the Fano factor $F = 0.16$. The number of created electrons is $N = E/\varepsilon = 5770$ and its uncertainty is $\Delta N = \sqrt{FN} = 30$. The uncertainty of the measured energy is

$$\Delta E = \frac{\Delta N}{N} E = \frac{\sqrt{F \cdot \frac{E}{\varepsilon}}}{\frac{E}{\varepsilon}} E = \sqrt{FE\varepsilon} = 0.8\,\mathrm{keV}\,.$$

For silicon, from another table we find $\varepsilon = 3.64\,\mathrm{eV}$ and $F = 0.115$, and by the same calculation as for the argon detector, we obtain the uncertainty $\Delta E = 0.25\,\mathrm{keV}$. The light yield of $LaBr_3(Ce)$ is $\Gamma = 63000$ photons/MeV. The number of created photons is $N = E\Gamma = 9450$. The problem statement said that the Fano factor should be taken equal to 1, so the uncertainty in the number of photons is $\Delta N = \sqrt{N} = 97$. The energy uncertainty is $\Delta E = (\Delta N/N)E = 1.5\,\mathrm{keV}$.

Subproblem 3

The total energy of the particle is

$$E = E_k + E_0 = \gamma E_0\,,$$

where E_k is the kinetic energy ($3\,\mathrm{GeV}$ for all these particles), E_0 is the particle's rest energy, and $\gamma = 1/\sqrt{1-\beta^2}$ is the particle's gamma factor. The rest energies of the electron, proton, antiproton, and helium nucleus are 0.5110 MeV, 938.3 MeV, 938.3 MeV, and 3727 MeV. The particle's beta factor (the ratio of its speed to the speed of light) is

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \left(\frac{E_0}{E_k + E_0}\right)^2}\,.$$

The beta factors for the considered electron, proton, antiproton, and helium nucleus are, respectively, 0.9999999855, 0.9712, 0.9712, and 0.8325. The momentum of the particle is

$$p = \gamma m v = \gamma \beta m c\,.$$

In a Cherenkov detector, a particle with speed $v > c/n$ produces a signal, which for NaF means $v > 0.7518c$ and for aerogel $v > 0.9524c$. We now discuss what signals we can expect from the individual particles.

- An electron with kinetic energy $E_k = 3\,\mathrm{GeV}$, speed $0.9999999855c$, and gamma factor 5872 will produce a very strong signal in the transition radiation detector, because the signal strength is proportional to $q^2\gamma$. It will also produce a signal in both Cherenkov detectors, where the two Cherenkov angles will be $\theta_{\mathrm{NaF}} = 41.2^\circ$ and $\theta_{\mathrm{aer}} = 17.8^\circ$.
- A proton with kinetic energy 3 GeV, speed $0.9712c$, and gamma factor 4.19 will produce a signal in the transition radiation detector that is several orders of magnitude weaker compared to the electron. It will also produce a signal in both Cherenkov detectors with angles $\theta_{\mathrm{NaF}} = 39.3^\circ$ and $\theta_{\mathrm{aer}} = 11.3^\circ$.
- An antiproton with kinetic energy 3 GeV will have the same signature as the proton in all three detectors. The only difference is bending to the opposite side in the magnetic spectrometer compared to the proton, since the antiproton has the opposite charge.
- A helium nucleus with kinetic energy 3 GeV, speed $0.8325c$, and gamma factor 2.24 will produce a slightly stronger signal in the transition radiation detector than the proton and antiproton, because the effect of the higher charge outweighs the lower gamma factor. Of the Cherenkov detectors, however, only NaF will be triggered, where the emission angle will be $\theta_{\mathrm{NaF}} = 25.4^\circ$.

Subproblem 4

The Bethe-Bloch equation was given in the text

$$-\frac{1}{\rho}\left\langle\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle = K z^2 \frac{Z}{A}\frac{1}{\beta^2}\left(\ln\left(\frac{2m_e c^2\gamma^2\beta^2}{I}\right) - \beta^2 - \frac{\delta(\beta)}{2} - \frac{C(\beta)}{Z}\right)\,.$$

For the purposes of this subproblem, we are interested only in the dependence on β and neglect the correction terms, so we rewrite it in the form

$$f(\beta) = -\frac{1}{\rho}\left\langle\frac{\mathrm{d}E}{\mathrm{d}x}\right\rangle \propto \frac{1}{\beta^2}\ln\left(A\,\gamma^2\beta^2\right) - 1 = \frac{1}{\beta^2}\ln\left(A\,\frac{\beta^2}{1-\beta^2}\right) - 1\,,$$

where $A = 11080$ is a constant. To obtain β for an MIP particle, we must differentiate the equation and set it equal to zero:

$$\frac{\mathrm{d}f}{\mathrm{d}\beta} = 0 = -2\frac{1}{\beta^3}\ln\left(A\ \frac{\beta^2}{1-\beta^2}\right) + \frac{2}{\beta^3(1-\beta^2)}\,,$$

$$\ln\left(A\frac{\beta^2}{1-\beta^2}\right) = \frac{1}{1-\beta^2}\,,$$

$$\beta = \sqrt{1 - \frac{1}{\ln\left(A\frac{\beta^2}{1-\beta^2}\right)}}\,.$$

We solve this equation iteratively and obtain $\beta = 0.9562$. An iterative numerical solution of equations requires that we express the equation with the unknown β in the form $\beta = f(\beta)$, which we did in the last line. We then substitute an initial estimate β_0 into the function $f(\beta)$, from which we obtain the next estimate of the solution $\beta_1 = f(\beta_0)$. Each further estimate β_{i+1} is obtained from the previous estimate β_i using the relation $\beta_{i+1} = f(\beta_i)$. If we are lucky, the results will converge to a single value, which is the desired solution of the equation. If we are unlucky and the values instead diverge, we must rearrange the equation and express β differently using another function $\beta = g(\beta)$. This function, fortunately, converges fairly quickly.

The kinetic energies $(\gamma - 1)mc^2$ of an electron, muon, and proton in the MIP regime are, respectively, 1.23 MeV, 255 MeV, and 2270 MeV.

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