Problem IV.1 ... running trees   3 points

Jarda is driving a car at a speed of $70\,\mathrm{km\cdot h^{-1}}$ and is approaching a car that has just entered a town and slowed down to $50\,\mathrm{km\cdot h^{-1}}$. Jarda sees the trees behind him in the reflection on the rear window of this car. What is the speed and direction of the trees' motion in the reflection relative to Jarda?   *Jarda's sample solution got away.*

First, we determine how fast the image of the trees formed by the rear window mirror moves relative to the ground, and then we transform this velocity into the frame associated with Jarda's car.

Because the rear window of the car is planar, the image of the trees behind this plane mirror is at the same distance from the glass as the glass is from the real trees. If the distance between the trees and the car window is x, then the distance between the window and the image of the trees is also x, so the distance between the image of the trees and the real trees is $2x$.

The car in front of Jarda moves with velocity $v_\mathrm{p} = 50\,\mathrm{km\cdot h^{-1}}$. After a time Δt, the distance between the car and the trees increases to $x + v_\mathrm{p}\Delta t$. The distance between the real trees and their image is twice as large, that is, $2x + 2v_\mathrm{p}\Delta t$; therefore, the image moves with velocity $2v_\mathrm{p}$ away from the trees in the direction of the car's motion.

Because the trees are at rest relative to the ground, their image moves with this velocity relative to the ground. If we subtract the velocity of Jarda's car, $v_\mathrm{J} = 70\,\mathrm{km\cdot h^{-1}}$, we obtain the relative velocity of the reflections of the trees and Jarda in the direction of motion of both cars as

$$v = 2v_\mathrm{p} - v_\mathrm{J} = 30\,\mathrm{km\cdot h^{-1}}\,.$$

With this velocity, the image of the trees in the rear window moves away from Jarda in the direction of motion of both cars.

Jaroslav Herman
jardah@fykos.org