Problem III.3 ... banana for scale

5 points

Lego was eating a banana. During the entire time he was eating, an unshielded gamma radiation source was located approximately 10 bananas away from him. This source irradiated Lego only while he was eating the banana, and was shielded at all other times. What is the activity of this source, given that it delivered the same radiation dose to Lego as the banana he ate? Estimate the properties of the bananas, the radiation source, and Lego, and express the result in SI units. *Lego realized that bananas are used for other things than size estimation.*

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where E is the amount of absorbed energy and m is the total mass into which the energy was absorbed. The second is the dose equivalent (denoted H), which modifies the dose by multiplying it by a quality factor. This factor determines the harmfulness of the given radiation for a human, and the newly defined quantity thus better represents how dangerous a dose an individual has received. By consuming one banana, we are exposed to $H = 10^{-7}\,\mathrm{Sv}$ (the so-called banana equivalent dose is even an informal unit for measuring radiation exposure). We want to determine the dose equivalent of gamma radiation with the same value. We therefore write

$$H = DQ = \frac{E}{m}Q\,,$$

where D is the gamma radiation dose absorbed by Lego, expressed as the radiation energy incident on Lego divided by its mass m, and Q is the quality factor of this radiation. One gamma photon has energy E_γ. The number of such photons emitted is $N = At$, where A is the sought source activity and t is the time during which the radiation was not shielded, that is, the time during which Lego ate the banana.

However, it is also necessary to realize that gamma rays are emitted uniformly in all directions; therefore, only the fraction corresponding to the area S_L occupied by Lego's body on a sphere with surface area $S_k = 4\pi r^2$ reaches Lego, where r is the radius given by the distance of Lego from the radiation source, that is, the length of 10 bananas. The desired expression for the radiation energy incident on Lego is

$$E_0 = E_\gamma At\frac{S_\mathrm{L}}{4\pi r^2}\,.$$

However, not all radiation will be absorbed by Lego. For irradiation,

$$E_\mathrm{pr} = E_0\mathrm{e}^{-\alpha\rho d}\,,$$

which gives the amount of gamma-ray energy incident on Lego (E_0) that passes through it (E_pr) and is not absorbed. This transmitted energy decreases exponentially with the thickness d (in our case, Lego's thickness), with the material density ρ through which the ray passes, and with the energy absorption coefficient α. All the energy that does not pass through Lego must remain in it; therefore, for the absorbed energy we have $E = E_0 - E_{\mathrm{pr}}$. This can be written as

$$E = E_0 - E_0 \mathrm{e}^{-\alpha\rho d} = E_0 \left(1 - \mathrm{e}^{-\alpha\rho d}\right) = E_\gamma A t \frac{S_\mathrm{L}}{4\pi r^2} \left(1 - \mathrm{e}^{-\alpha\rho d}\right) ,$$

and after substituting into the original relation,

$$H = \frac{E_\gamma A t S_\mathrm{L} \left(1 - \mathrm{e}^{-\alpha\rho d}\right)}{4\pi r^2 m} Q ,$$

solving for the activity yields

$$A = \frac{4\pi r^2 H m}{E_\gamma t S_\mathrm{L} Q \left(1 - \mathrm{e}^{-\alpha\rho d}\right)} .$$

Finally, let us make several estimates. Let us choose Lego's mass $m = 70\,\mathrm{kg}$, the area covered by its body $S_\mathrm{L} = 1\,\mathrm{m}^2$ (the total surface area of a human body is about $2\,\mathrm{m}^2$; since Lego is exposed to the source with half of its body, the absorption area will be half of the total human surface area). It eats the banana for $t = 60\,\mathrm{s}$; the length of one banana is $20\,\mathrm{cm}$, thus $r = 2\,\mathrm{m}$.

For simplicity, assume that Lego is everywhere one and a half bananas wide, that is, $d = 0.3\,\mathrm{m}$, and that its body consists homogeneously predominantly of muscle tissue with density $\rho = 1\,060\,\mathrm{kg{\cdot}m^{-3}}$. The gamma radiation quality factor is $Q = 1$, and for our gamma ray we take one emitted in the decay of barium-137, which has energy $E_\gamma = 662\,\mathrm{keV}$.

For this energy and muscle tissue, we find the value of the energy absorption coefficient $\alpha = 3.25 \cdot 10^{-3}\,\mathrm{m^2{\cdot}kg^{-1}}$ (we consider the coefficient for radiation with energy $600\,\mathrm{keV}$ from `https://www.physics.nist.gov/PhysRefData/XrayMassCoef/ComTab/muscle.html`). After substitution, we obtain

$$A \doteq 86\,\mathrm{MBq} ,$$

which in the case of our barium-137 would correspond to a small amount of 4 picograms. where E is the amount of absorbed energy and m is the total mass into which the energy was absorbed. The second is the dose equivalent (denoted H), which modifies the dose by multiplying it by a quality factor. This factor determines the harmfulness of the given radiation for a human, and the newly defined quantity thus better represents how dangerous a dose an individual has received. By consuming one banana, we are exposed to $H = 10^{-7}\,\mathrm{Sv}$ (the so-called banana equivalent dose is even an informal unit for measuring radiation exposure). We want to determine the dose equivalent of gamma radiation with the same value. We therefore write

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