Problem II.3 ... a physicist and a ball

5 points

A young soccer player finds that after an unfortunate shot, his football becomes wedged between two vertical parallel walls separated by a distance of 21*cm. The boy can exert a maximum force of* 450 N*, while the coefficient of friction between the ball and the walls is* 0.6*.*

However, being a participant of VYFUK physics competition, he knows quite a bit about physics. He therefore begins to spray the ball with cold water at a temperature of 10 °C*.*

Will he be able to pull the ball out afterwards? The ball has a radius of 11 cm*, and the contact areas with the walls are circular. Assume that the volume of the ball remains constant, both during its deformation between the walls and during cooling. Before the shot, the ball was inflated to* 120 kPa *and had a temperature of* 44 °C*. Jarda usually misses the goal too.*

The ball initially had a diameter of $2R = 22\,\mathrm{cm}$, but it has now deformed such that the radius of contact with the walls is given by:

$$r = \sqrt{R^2 - \frac{d^2}{4}} \doteq 3.3\,\mathrm{cm}\,,$$

where $d = 21\,\mathrm{cm}$ is the distance between the walls. The area of contact on either side is given as

$$A \ = \pi r^2 = \pi\left(R^2 - \frac{d^2}{4}\right)\,.$$

The change in pressure inside the ball due to the volume change is neglected. The normal force exerted by the compressed ball on the walls is

$$F_\mathrm{N} = 2A\ p = 2\pi r^2 p\,,$$

where p is the internal pressure of the ball. The boy must overcome the frictional force

$$F_\mathrm{t} = f F_\mathrm{N} = 2\pi f r^2 p\,.$$

To overcome this, he needs to reduce the internal pressure by cooling the gas inside the ball. From the ideal gas law, assuming constant volume and number of particles, we have

$$\frac{p_1}{T_1} = \frac{p}{T}\,,$$

where $T = 44\,^\circ\mathrm{C} \doteq 317\,\mathrm{K}$ is the initial temperature, $T_1 = 10\,^\circ\mathrm{C} \doteq 283\,\mathrm{K}$ is the lowest temperature the ball can reach, and p_1 is the pressure after cooling. The frictional force that the boy must overcome after cooling the air in the ball is

$$F_\mathrm{t} = f F_\mathrm{N} = 2\pi f r^2 p \frac{T}{T_1} \doteq 430\,\mathrm{N}\,.$$

Thus, the boy will be able to pull the ball out.

Jaroslav Herman
jardah@fykos.org

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