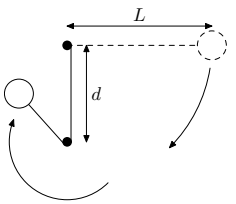
Problem II.2 ... a rod and a string

3 points



A mass point is attached to a thin horizontal rod by a massless, inextensible, flexible string of length L. The mass is initially placed so that the string is horizontal and perpendicular to the rod. The string is then taut and the mass is released downwards. At a distance $d < L$ below this rod, another identical rod is placed, parallel to the first. The mass begins to wrap around the lower rod as it moves. What is the minimum distance d between the rods such that the mass completes a full revolution around the lower rod while the string remains taut at all times?

Jarda would like to sit on such a swing.

For circling around the lower rod, the following equation holds

$$r = L - d\,.$$

In order for the string to always be taut, the centrifugal force must be greater than the gravitational force at the highest point of the circle trajectory of the point mass, so a boundary condition is

$$\frac{mv^2}{r} = mg \quad \Rightarrow \quad v^2 = gr\,.$$

We can find the speed of the point mass from the law of conservation of energy

$$mg(L-2r) = \frac{1}{2}mv^2 \quad \Rightarrow \quad v^2 = 2g\,(L-2r)\ ,$$

which leads to the equation

$$2g\,(L-2r) = gr \quad \Rightarrow \quad r = \frac{2}{5}L\,,$$

so the distance we are looking for is

$$d = L - r = \frac{3}{5}L\,.$$

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