Problem I.E ... lost object 11 points

Sometimes, maybe even some of you are wondering how far a small item can get right after it slips from your hands and falls to the ground. Choose a small item (screw, nail, and so on) and measure the distance between the place where you dropped the item and the place where you have found it, in relation to the height (from the ground) from which you dropped it. For each initial height, repeat the measurement enough times to decrease the statistical error of your measurement. Think about suitable surfaces and items.

Jarda cracked when the screws and nuts fell from his hands again.

Introduction and Theory

Let us attempt to construct the simplest possible model that can help us describe the situation and compare it with the measured results. How far from the point of impact can the object travel? This is determined by its initial horizontal velocity v_h and the friction coefficient f acting between the object and the surface. The kinetic energy is transformed into work done by the frictional force according to the law of conservation of energy

$$\frac{1}{2}mv_h^2 = mgfd\,,$$

where m is the mass of the object, g the gravitational acceleration, and d the distance traveled. Already at this stage, our model deviates from reality, as it neglects the object's rotation, bouncing, and other complex motions. Including these effects would, however, complicate the situation considerably.

What is the initial horizontal velocity? From the law of conservation of energy, we can find the vertical velocity v_v just before impact:

$$\frac{1}{2}mv_v^2 = mgh\,,$$

where h is the initial height of the object above the surface. Upon impact, both the magnitude and direction of the velocity change, but we may assume that the velocity after the bounce is proportional to the velocity before impact, with some proportionality coefficient k:

$$v_h = k\ v_v\,,$$

where k depends on the geometry of impact, the material of the object, and many other parameters. Combining the above assumptions, we obtain a relationship between the initial height and the distance d:

$$mgh = \frac{1}{2}mv_v^2 = \frac{1}{2}m\frac{v_h^2}{k^2} = \frac{mgfd}{k^2}\,,$$

$$\frac{k^2}{f}h = d\,.$$

According to our highly simplified model, the distance from the point of impact should therefore be proportional to the height from which the object was dropped.

Measurement Procedure

As a test object, a small hexagonal nut from the *Merkur* construction set was used. The height from which the nut was dropped by hand was measured with a folding ruler. At the beginning, the nut was held in a random orientation to simulate the situation described in the problem statement—dropping an object from a random position. The nut was always released above a marked point that defined the impact location.

For each height, 20 nuts were dropped. After each drop, the nut was not immediately removed; instead, the next one was dropped. Only after all nuts had been released was the surface photographed, showing all the final positions of the nuts. One of the resulting photographs is shown in Figure 1. The nuts were spaced sufficiently apart so that they did not collide with each other, allowing this procedure to be used.

Figure 1: Photograph of the measurement for the same height h, from which the data were later extracted.

The distances of the nuts from the impact point were measured using the program *GeoGebra*. As a reference for conversion to real dimensions, the side length of a floor tile on which the experiment was performed was used. This length was measured as 30 cm.

Results

The measured distances d from the impact point for various heights h are listed in Table 1. The range of heights was limited by the dimensions of the room in which the measurement took place.

We observe that the values vary significantly even for the same height. Thus, the result strongly depends on the impact conditions. Nevertheless, we compute the mean value $\bar{d}$ that the object traveled, along with the standard deviation according to

$$\sigma_{\bar{d}} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(d_i - \bar{d}\right)^2}\,,$$

where $N = 20$ is the number of measurements for each height and d_i are the individual measured values. The corresponding results are listed in Table 2 and plotted in Graph 2. We also calculate the relative error $\sigma_r = \sigma/\bar{d}$. Since the scatter of measured values is much larger than the precision of measurement, the uncertainty of distance measurement itself can be neglected.

Table 1: Nut displacement d from impact point vs. drop height h.

$\frac{h}{\text{cm}}$	5	10	15	20	25	30	35	40	50	60
Nut	$\frac{d}{\text{cm}}$	$\frac{d}{\text{cm}}$	$\frac{d}{\text{cm}}$	$\frac{d}{\text{cm}}$	$\frac{d}{\text{cm}}$	$\frac{d}{\text{cm}}$	$\frac{d}{\text{cm}}$	$\frac{d}{\text{cm}}$	$\frac{d}{\text{cm}}$	$\frac{d}{\text{cm}}$
1	10.3	20.4	19.4	63.3	30.5	62.4	67.7	35.6	71.9	81.7
2	16.7	29.8	20.6	36.3	34.8	56.0	54.9	53.5	70.1	100.3
3	9.7	20.0	16.6	37.8	43.1	51.2	36.7	55.0	56.1	93.5
4	8.3	9.4	24.4	31.1	56.4	50.5	58.7	39.1	72.5	86.1
5	8.2	21.0	24.2	30.4	57.8	42.6	71.8	41.5	60.0	60.2
6	5.6	15.4	22.6	24.3	45.5	41.9	64.1	48.2	58.2	93.5
7	4.7	11.5	22.2	25.7	32.1	61.7	40.2	59.7	64.8	71.9
8	6.5	10.9	26.1	15.2	17.5	36.0	37.0	53.2	45.4	102.4
9	7.3	8.0	21.9	19.6	32.6	32.4	28.4	34.7	49.0	72.2
10	5.9	7.0	10.3	17.8	33.1	38.6	42.4	38.5	46.6	46.0
11	4.1	3.7	13.3	14.8	29.5	22.6	39.6	41.5	53.5	59.7
12	5.6	4.6	12.9	13.7	27.9	24.5	33.8	37.4	41.9	58.2
13	2.8	7.4	8.9	16.3	26.0	20.0	31.0	30.6	46.3	40.7
14	3.4	8.3	9.4	8.5	14.3	20.5	32.2	21.8	38.3	48.0
15	4.5	5.4	11.4	8.7	22.7	20.5	33.5	14.1	31.5	23.3
16	5.0	5.2	10.1	7.4	14.9	20.5	14.0	20.9	29.1	25.0
17	4.6	6.3	11.7	10.7	8.6	24.5	21.4	15.6	32.7	15.9
18	2.5	5.2	11.9	1.5	6.7	7.9	22.7	17.1	19.9	37.2
19	2.0	3.9	10.9	3.5	1.2	8.8	7.0	11.8	29.1	35.5
20	1.0	4.3	8.5	35.7	85.7	41.9	54.6	3.8	44.6	15.9

Table 2: Mean distance $\bar{d}$ of the nut from the impact point after being dropped from height h, standard deviation σ, and its relative value σ_r.

$\frac{h}{\text{cm}}$	$\frac{\bar{d}}{\text{cm}}$	$\frac{\sigma}{\text{cm}}$	$\frac{\sigma_\text{r}}{\%}$
5	6	3	58
10	10	7	68
15	16	6	37
20	21	14	69
25	31	19	63
30	34	16	47
35	40	17	44
40	34	16	47
50	48	15	31
60	58	28	47

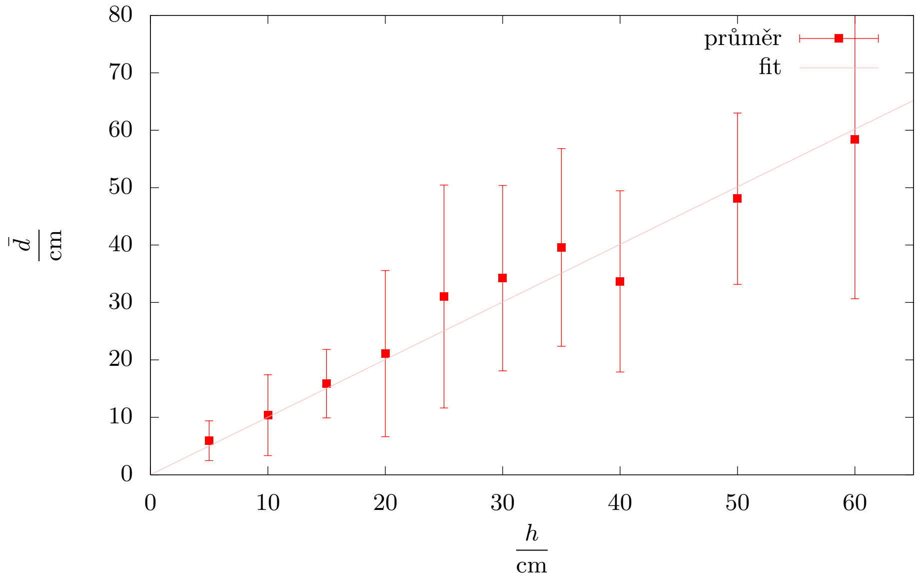

Figure 2: Graph of the dependence of the mean distance from the impact point on the drop height.

We see that the mean distance $\bar{d}$ increases linearly with height h, consistent with our simple theoretical model. The data were fitted with a straight line using *Gnuplot*:

$$f(x) = a\ x$$

and the fit parameter was obtained as

$$a\ = \frac{k^2}{f} = 1.00 \pm 0.04\,.$$

According to our measurement, the nut on a flat surface travels, on average, the same distance as the height from which it was dropped.

Analysis of results

For each height, 20 measurements were performed. Different nuts were used, but this should not affect the outcome, as they are assumed identical. The dominant source of variation is the initial position and velocity at release, as well as the details of the impact. The floor was composed of tiles separated by thin gaps, in which the nut could get stuck; such trials were repeated. The photographic measurement method may introduce systematic errors due to geometric distortion.

Within a single height, Table 1 shows large variations—the ratio between the smallest and largest distances can reach several tens. Clearly, the result depends strongly on the initial conditions determining the impact angle and velocity. The large scatter is consistent with the intention to hold the nut in a random orientation. Some nuts barely moved after impact, while others traveled farther than the drop height. We can therefore assume that our measurements covered nearly the full range of possible outcomes, fulfilling the requirement of random orientation.

In further analysis, we computed the mean and standard deviation and plotted them in a graph, obtaining a linear dependence consistent with theory. Although high precision was not expected, the relative error of the fitted parameter a is only 4 %, indicating good agreement with a linear relationship. Even with a simple model, we thus arrived at a qualitatively correct result.

owever, the error of parameter a does not account for the large error bars. These are quite substantial, with relative errors in the tens of percent, as is already evident from the wide scatter of the values. It would be preferable to employ a linear fitting method that incorporates these error bars into the final uncertainty of the parameter. Furthermore, our entire approach of using the arithmetic mean is not entirely suitable for this task. We cannot search for a single specific value because the result depends heavily on the initial configuration of the nut upon release. More accurately, we should be looking for a probability distribution of the distance the nut reaches when dropped randomly.

Such a distribution would take the form of a function, from which we could identify, for example, the most probable or median value. To determine this function for each height, however, we would need to perform many times more measurements. During data processing, we attempted to plot distance histograms for individual heights to approximate this function; unfortunately, we did not achieve a satisfactory or publishable result. Nevertheless, since the relative error does not change significantly with height h (Table 2), we can at least infer that the distribution might have a similar shape across different heights. Outliers are typically removed if they deviate by more than 3σ, as they are unlikely and may result from measurement errors. However, since σ itself is large in our case, this procedure would not eliminate any data points.

Our measurement yielded $a = k^2/f = 1.00 \pm 0.04$, implying that the nuts, on average, traveled the same distance as the drop height—a seemingly interesting result. The exact value, however, depends on the specific combination of object and surface; different conditions would yield different a values.

Finally, we can conclude that the chosen combination of object and surface was appropriate for our experiment. The nuts bounced to large distances, sometimes exceeding the drop height. If the experiment were conducted on sand, for example, no measurable motion would occur. A hard, flat, and level surface is thus essential, as any slope would introduce additional gravitational components. The area must also be large enough to avoid interference from obstacles.

The object should satisfy similar requirements of hardness and elasticity as the surface. A piece of modeling clay, for example, would not move at all after impact. The large scatter of values for each height suggests that the object was not perfectly symmetrical, as d varied greatly with its initial orientation.

Conclusion

We measured the mean distance $\bar{d}$ at which a dropped object comes to rest as a function of the drop height h. According to theory, the data were fitted with a straight line, yielding for our object–surface pair $\bar{d} = 1.00 \pm 0.04h$, confirming that the theoretical model is valid to first approximation. We also discussed the suitability of the data processing and the interpretation of the results.

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