Problem I.P ... the most efficient drive

10 points

Determine the most efficient drive for a car. More exactly, determine such a drive, which consumes the least amount of energy per 1 J of work done by the engine, considering the entire process from fuel production to engine efficiency. You can compare options such as gasoline, diesel, electricity, hydrogen, or feed for a harnessed horse.

Jarda wanted to deliver the philodendron effectively.

The author of the solution is Daniel Švec.

Introduction

We consider a powertrain to be one that is capable of mass distribution similar to that of today's conventional engines.

Horse (animals)

We will not take into account the added value that animal husbandry brings. To spare you the consideration of basal metabolism and the efficiency of the horse's digestive tract, we will directly take into account its approximate daily food intake and the work done. The sources consulted indicate that a healthy adult horse doing heavy work must have a food intake of up to 4 % of its weight. In view of the necessity of pulling a carriage, it may be assumed that the strongest breeds, which will not require such a number of individuals, will be the most suitable. These would include, for example, the Clydesdale, the Scythian, the Percheron and the Belgian. The average weight of these breeds is between 900 and 1 200 kg. If we consider one carriage pulled by a two-horses team for a household (estimate for a family of 4), with the horses taking turns in two shifts, we get the necessary number of horses equal to the number of people, i.e. approx. $8 \cdot 10^9$. The harvest necessary to feed the horses must therefore be brought forth:

$$m = \frac{4}{100} m_\mathrm{k} t N = \frac{4}{100} \cdot 1\,200 \cdot 365.25 \cdot 8 \cdot 10^9\,\mathrm{kg} \doteq 1.4 \cdot 10^{14}\,\mathrm{kg}\,,$$

Considering that wheat is produced in the Czech Republic at $5.5 \cdot 10^9$ kg per year, it would be necessary to increase wheat production worldwide by twenty-five thousand times the Czech production. This involves additional associated costs of cultivation which, moreover, cannot be realised anywhere in the world because of unsuitable conditions. In addition to the care of the feed itself, fertilisation and spraying must also be considered. Last but not least, it cannot be ruled out that a marked increase in cereal consumption may result in a shortage of food for humans, which may be reflected in food prices. Horses also require some veterinary care and it is likely that most of the time they would not be able to be used to their full potential – because of the sleeping and resting needs that machines do not require. From these considerations, it follows that animals cannot be considered both reliable and the most efficient means of propulsion. Humans, however, do not desire the most efficient drive, but a sufficient drive. The most self-serving imaginary animal with an unrealistic efficiency of 100% is useless if it can do 1 J of work per day.

Petrol

Gasoline, like diesel, is derived from petroleum. It must primarily be extracted by drilling, but first the rock itself must be removed. The oil then either flows out on its own or must be extracted – since we are neglecting here the efficiencies of the techniques used, we will consider here the extraction of oil rather than its considerably less energy-intensive autonomous recovery. Next, we estimate the density of oil to be $\rho_2 = 1\,\mathrm{kg{\cdot}dm^{-3}}$ (mean of $0.8 - 1.2\,\mathrm{kg{\cdot}dm^{-3}}$), the density of rock to be $\rho_1 = 5\,500\,\mathrm{kg\cdot m^{-3}}$ (the density of the Earth), and the borehole to be a cylinder with radius $r = 1\,\mathrm{m}$ and height of $100 - 10\,000\,\mathrm{m}$, i.e., approximately $h = 4\,000\,\mathrm{m}$ on average. In total, we use $N = 6$ wells per reservoir, and this contains an estimated $V_r = 150\,000$ barrels ($1\,\mathrm{barel} = 159\,\mathrm{l}$) of oil in addition to the gas (which we do not now consider). It is also important to remember that the production of an engine or an automobile itself is both energy and raw material intensive. For simplicity, we include these costs in the extraction demandingness as half of the energy/labor required to extract the rock (not the oil). In terms of extracting the rock, note that on average it must be raised by $h/2$, since the first part has height 0 and the last part has full depth h.

$$\begin{aligned} W_1 &= Nm\frac{h}{2}g = N\rho_1 V g\frac{h}{2}\,, \\ &= N\rho_1\pi r^2 hg\frac{h}{2}\,, \\ &= 3\pi\rho_1 g r^2 h^2\,. \end{aligned}$$

For oil production, we will assume that its reserves are located at approximately the same depth (denoted as h above), which (between the bottom and the surface of the oil) is negligible compared to the height of the cylinder used to extract it:

$$W_2 = mgh = \rho_2 V_r gh\,.$$

For the work to produce the engine and the car itself, the following applies

$$W_3 = \frac{1}{2}W_1\,.$$

The total need is thus

$$\begin{aligned} \sum W &= W_1 + W_2 + W_3\,, \\ &= W_1 + W_2 + \frac{1}{2}W_1\,, \\ &= \frac{3}{2}\cdot 3\pi\rho_1 g r^2 h^2 + \rho_2 V_r gh\,, \\ &= \frac{9}{2}\pi\cdot 5\,500\cdot 9.81\cdot 1^2\cdot 4\,000^2 + 1\cdot 150\,000\cdot 159\cdot 9.81\cdot 4\,000\,\mathrm{J} \doteq 1.3\cdot 10^{13}\,\mathrm{J}\,. \end{aligned}$$

This means that per $1\,\mathrm{l}$

$$1\,\mathrm{l} = \frac{1.3\cdot 10^{13}\,\mathrm{J}}{150\,000\cdot 159} \doteq 550\,\mathrm{kJ}\,.$$

We know that the average consumption of a car with a petrol or diesel engine is $8\,\mathrm{l} - 10\,\mathrm{l}$ for $100\,\mathrm{km}$– we use a higher figure. A car moves on wheels, and the largest wheels that can be fitted to some cars have a radius of about $r = 30\mathrm{cm}$, so we have to consider the rolling resistance (for a wheel on asphalt $\xi = 0.5\,\mathrm{mm}$)

$$rF = \xi F_G \quad\Rightarrow\quad F = mg\frac{\xi}{r}\,,$$

$$W = Fs = 1\,\mathrm{J} \quad\Rightarrow\quad s = \frac{Wr}{mg\xi}\,.$$

Fuel consumption is directly proportional to the distance s

$$\mathrm{Sp} = ks = \frac{kWr}{mg\xi}\,.$$

where k indicates how many liters of gasoline the car uses per 100 km. Including engine efficiency ($\eta = 20 - 40\,\% \to 35\,\%$)

$$\mathrm{Sp}' = \frac{k}{\eta}\,,$$
$$s = \frac{k\;Wr}{m\eta g\xi}\,.$$

Since consumption is given in v l/100 km, we can keep litres, but due to the use of SI base units we have to convert the denominators to metres

$$100\,\mathrm{km} = 100\,000\,\mathrm{m} = 10^5\,\mathrm{m}\,.$$

So we must divide the equation by 10^5 (multiply by 10^{-5}):

$$\begin{aligned} V &= \frac{k}{\eta}\cdot 10^{-5}\cdot\frac{Wr}{mg\xi}\\ &= \frac{10}{0.35}\cdot 10^{-5}\cdot\frac{1\cdot 30}{1\,200\cdot 9.81\cdot 0.05}\,\mathrm{l}\\ &\doteq 1.456\cdot 10^{-5}\,\mathrm{l} \doteq 1.5\cdot 10^{-5}\,\mathrm{l} \doteq 0.015\,\mathrm{ml}\,. \end{aligned}$$

If we know that we need 550 kJ of energy for 1 l, we can calculate how much energy will be needed for 0.015 ml.

$$E = 550\,000\cdot\frac{0.015}{1\,000}\,\mathrm{J} \doteq 8.25\,\mathrm{J} \doteq 8\,\mathrm{J}\,.$$

Electric Vehicle

An electric car uses straight electricity to move, the conversion of which is not so demanding, and thus achieves an efficiency (95 %) incomparable to previous cases. Before proceeding to considerations of the production of electricity, we consider the energy balance of motion itself. A certain disadvantage here is the variety and difference of different models of personal electric vehicles, which makes it impossible to determine an approximate pattern. Thus, it cannot be excluded that the conclusions drawn are not generally valid. Again, the determination of rolling resistance and the resulting quantification of the trajectory apply here. Further, however, the steps differ slightly from a gasoline or diesel car because the performance or efficiency of an electric car is characterized by its range. For a given distance (usually 100 km), there is a corresponding energy consumption, so the calculations are simplified. As an example, let's take the Tesla Model S, a relatively well-sold car according to sources. Its weight should be approximately 2 100 kg and its energy consumption 24 kWh per 100 km. In this respect one may be skeptical of these figures , as the next figure talks about a battery with 85 kWh capacity for 426 km. So, for 1 km, there is

$$a_1 = \frac{24}{100}\,\mathrm{kWh{\cdot}km^{-1}} = 0.24\,\mathrm{kWh{\cdot}km^{-1}}\,,$$
$$a_2 = \frac{85}{426}\,\mathrm{kWh{\cdot}km^{-1}} = 0.20\,\mathrm{kWh{\cdot}km^{-1}}\,.$$

This contradiction (difference 20 %) can be explained either in terms of physics or chemistry as uneven battery performance depending on its charge, or as an advertising omission by the company for more attractive results. For practicality and simplicity of calculations we will consider a constant battery power and the originally stated consumption values

$$rF = \xi F_G \quad \Rightarrow \quad F = mg\frac{\xi}{r}\,,$$
$$W = Fs \quad \Rightarrow \quad s = \frac{W}{F} = \frac{Wr}{mg\xi}\,,$$

and then

$$\mathrm{Sp} = sk = \frac{k\,Wr}{mg\xi}\,,$$
$$\mathrm{Sp}' = E = \frac{Sp}{\eta} = \frac{k}{\eta}\frac{Wr}{mg\xi}\,,$$

where

$$\frac{24\cdot 10^3\cdot 3\,600}{0.95}\cdot\frac{1\cdot 30}{2\,100\cdot 9.81\cdot 0.05}\cdot 10^{-5}\,\mathrm{J} = 26.488\,\mathrm{J} \doteq 27\,\mathrm{J}\,.$$

As can be seen from the results, an electric car consumes 27 J per 1 J work done, about three times more than a gasoline or diesel car.

For this reason, it makes very little sense (from a purely energy point of view) to consider the most efficient way of harvesting electricity when the electricity itself is not the most efficient as such.

Of course, it cannot be ruled out that the electric car is not dependent on fossil fuels (lithium only). We will ignore the issues of lithium mining for battery production, as well as electricity generation in coal, nuclear (stable), solar, tidal (unstable) power plants.

Based on these additional associated but absolutely necessary costs, one can imagine that the total energy consumption of EVs would be many times higher.

I also found remarkable the claim that the consumption of an electric car (24 kWh/100 km) is roughly equivalent to 2.6 l gasoline.

This statement, if interpreted as consumption, would certainly make the electric car a legitimately attractive alternative with up to five times lower operating costs.

The second way of interpreting it speaks of energy equality if such amount of gasoline were burned. It has a calorific value of $40\,\mathrm{MJ\cdot kg^{-1}}$ and a density of $750\,\mathrm{kg\cdot m^{-3}}$

$$E_1 = E_2\,,$$
$$24\,\mathrm{kWh} = Hm = H\rho V\,,$$
$$V = \frac{24\,\mathrm{kWh}}{H\rho} = \frac{24\cdot 10^3\cdot 3\,600}{40\cdot 10^6\cdot 750}\,\mathrm{m^3} = \frac{24\cdot 3.6}{40\cdot 750}\,\mathrm{m^3} \doteq 2.8\cdot 10^{-3}\,\mathrm{m^3} = 2.8\,\mathrm{l}\,,$$

which is close to the reported value 2.6 l.

This reference to the alleged consumption advantage of the electric car over gasoline is thus false, since it does not compare the energy required to do the chosen work, but the ideal state at full energy utilization, which the principle of the internal combustion engine does not even aspire to: the injected fuel is utilized by adiabatic action and thus uses energy less efficiently than simple combustion, but still more efficiently than the electric car.

Hydrogen

A hydrogen car uses liquefied hydrogen to power it, which can either be burned, which is less common, or react with oxygen.

The basic principles of the car's motion are very similar to those of an electric car, here the car reaches a weight of 1.8 t, the engine has an efficiency of up to 60 % and the tank can hold up to 6.3 kg of hydrogen, on which it should travel 600 km – 670 km. For the sake of practicality, let's set this value at 630 km. I have chosen the Hyundai Nixa as the role model for these specifications:

$$F = \frac{mg\xi}{r} \quad \Rightarrow \quad s = \frac{Wr}{mg\xi}\,,$$

$$\mathrm{Sp} = k\frac{Wr}{mg\xi} = \frac{6.3\cdot 1\cdot 30}{630\cdot 1\,800\cdot 9.81\cdot 0.05}\,\mathrm{g} = 3.4\cdot 10^{-4}\,\mathrm{g} = 3.4\cdot 10^{-7}\,\mathrm{kg}\,.$$

Next, we consider that all of this hydrogen will be burned, so should release the heat of combustion (if the reaction is isobaric): $H = 142\,\mathrm{MJ\cdot kg^{-1}}$. In this case, we choose to make the heat transfer mass-dependent, since a certain amount of hydrogen (if it does not escape) is characterized by the number of molecules, each of which has a mass. Since the gas can be both heated/cooled and compressed, the calculation with mass can be considered more objective and independent.

Burning this amount of hydrogen will ideally release a value of energy:

$$E = Hm = 142\cdot 10^{6}\cdot 3.4\cdot 10^{-7}\,\mathrm{J} \doteq 48.3\,\mathrm{J}\,.$$

Of this amount, however, the car practically consumes and uses only a certain part, the upper limit of which corresponds to the maximum efficiency:

$$E' = \eta E = 0.6\cdot 48.3\,\mathrm{J} \doteq 29\,\mathrm{J}\,.$$

This value is quite close to the result of an electric car, similarly to it. Now we will make a rough consideration of the associated costs –these do not need to be specified, as these fuels come out less favourably than petrol anyway.

Certain expenses can be expected for the production (either from non-renewable natural gas or by electrolysis)

$$m = \frac{MIt}{FV}\,,$$

$$t = \frac{mFV}{MI}\,,$$

$$W = UIt = \frac{UImFV}{MI} = \frac{mFVU}{M}\,.$$

This is the energy needed to release hydrogen of m mass by electrolysis, where U and I denote the voltage and current at which electrolysis takes place, M is the molar mass of hydrogen, F is Faraday's constant, and V is the number of electrons that hydrogen in a given molecule lacks until it reaches electrical neutrality.

We also know that for practicality and efficiency of transport, this gas is both compressed and liquefied (both by pressure and by change of state). Compression cannot be isochoric in principle, the work can be

$$W_{\mathrm{bar}} = mc_p\Delta T$$

for isobaric compression,

$$W_{\mathrm{term}} = p_1 V_1 \ln \frac{V_1}{V_2}$$

for isothermal compression,

$$W_{\mathrm{ad}} = \frac{5}{2}(p_2 V_2 - p_1 V_1)$$

for adiabatic compression, where p_1, V_1, and p_2, V_2 are the pressure and volume of the gas at the initial and end state, respectively, and c_p is the specific heat capacity of the gas at constant pressure.

For the phase change (from gaseous to liquid)

$$L_{\mathrm{v}} = m l_{\mathrm{v}} = m \cdot 454\,\mathrm{kJ \cdot kg^{-1}}\,,$$

where l_{v} is the specific latent heat of boiling of hydrogen.

It must also be considered that the boiling point of hydrogen is not very high – about 20 K, so that the drop in temperature or change in pressure must be significant.

If, after compression, there is still some temperature to reach the boiling point corresponding to the given pressure, energy must still be removed

$$W = mc\Delta T\,.$$

Another disadvantage of hydrogen is its reactivity with oxygen, where it explodes. For this reason, it is necessary to keep them strictly separated during storage and transport, which further increases the overall cost.

As can be seen from the well-known example of airships, apart from its flammability and certain unpredictability, hydrogen is also characterized by very low density and light molecules whose continuous leakage is almost impossible to prevent.

In this respect, hydrogen can clearly be considered less energy efficient than petrol.

Steam Machine

Although the steam engine could easily be described as obsolete and clearly unsuitable for the passenger car (and due to the amount of smoke falling on other cars, whose visibility it reduces, impractical and dangerous), we will try to prove the same by reasoning and calculations.

The usual efficiency of a steam engine is in the 9 % – 15 % range, which in itself is very low when the internal combustion engine reaches values more than twice as high, and the electric motor over 90 %.

Efficiency alone prescribes the necessity of getting to the engine or boiler 1 J to work

$$\eta = \frac{W}{W_0}\,,$$

$$W_{0_{\min}} = \frac{W}{\eta} = \frac{1}{0.15}\,\mathrm{J} = 7\,\mathrm{J}\,,$$

$$W_{0_{\max}} = \frac{W}{\eta} = \frac{1}{0.09}\,\mathrm{J} = 11\,\mathrm{J}\,.$$

Here, the calorific value and quality of the coal being burned must also be taken into account. Also, the extraction of the coal itself (whether in the quarry or in the shafts) is certainly energy intensive. Gasoline has reached a value of 8 J ($8\,\langle 7, 11\rangle$), so it is safe to expect that the cost of coal extraction will outstrip even the considered production of an automobile –after all, so did the value at 1 J with lower efficiency.

The steam engine can thus (at least in its contemplated form, if it does not undergo renovations and innovations) be considered an unsuitable way of powering a passenger car.

Wind

In general, the most efficient propulsion can be considered to be that which has the lowest cost of both the production of the "engine" itself (or the boiler and similar key devices used to convert one type of energy to another) and the energy gain.

From this point of view, renewables are the most advantageous – unlike the still sovereignly leading gasoline, they are not limited by finite quantities. Of these, wind appears to be the most useful. The energy of water and solar radiation is also almost unlimited, but highly variable. A more limiting factor, however, is the fact that this energy must be converted into usable energy mainly through electrical energy, the shortcomings of which have been described above. For the automobile, the power of water is hardly usable, and solar energy also requires rather expensive (silicon) panels for conversion, which are limited by the availability of material.

While geothermal energy is usable for individual charging stations, it cannot be used while driving, so it is dependent on the ability to recharge. Biomass combustion is linked to a sufficient supply of biomass and is unlikely to be a more environmentally friendly source.

So wind successfully avoids all these pitfalls – it is available to the car-ship all the time and its extraction does not constitute any added work, which would have to be accounted for in the overall energy balance.

The auto-ship, a completely fictional kind of land vehicle, resembles the frame of an automobile, which also has a mast and (for ease of thought) a single sail. The cost of making the sail (both energy and material) can reasonably be considered negligible compared to the cost of making solar panels (where precision work and metallic mining are required).

We also know that the car-boat is set in motion by the wind force acting on the sail with S_1 area and C_1 drag coefficient; in contrast, the drag force acts on the profile of the car-boat, whose area we denote by S_2 and C_2 drag coefficient (but also partly on the sail, which we replace by a double force on the profile), and the rolling resistance of the tyres.

In order for the car-ship to move, the lowest equality of the thrust force F_{thr} and the air drag force F_{dra} and frictional F_{fr} is required. For the wind speed v we will consider a windless range up to $15\,\mathrm{m\cdot s^{-1}}$, since these values would normally occur according to the Beaufort scale

$$F_{\mathrm{thr}} = 2F_{\mathrm{drag}} + F_{\mathrm{fr}}\,,$$

$$\frac{1}{2}C_1 S_1 \rho v^2 = 2\cdot\frac{1}{2}C_2 S_2 \rho v^2 + mg\frac{\xi}{r}\,,$$

$$S_1 = \frac{2C_2}{C_1}S_2 + \frac{2mg\xi}{rC_1\rho v^2}\,,$$

where ρ denotes the density of air.

To ensure that the results are not distorted by over-optimistic reasoning, we will consider the mass of the auto-ship $m = 1\,500\,\mathrm{kg}$. This exaggeration perhaps sufficiently compensates for the simplifications made (e.g., the perfect impermeability of the sail).

We estimate the drag coefficient for the sail to be $C_1 = 1.2$ and for the hull of the auto-ship $C_2 = 2.0$, with a width of $a = 1.7\,\mathrm{m}$ and a height of $b = 1.5\,\mathrm{m}$. The rolling resistance arm is again $\xi = 0.05\,\mathrm{cm}$ and the tire radius $r = 30\,\mathrm{cm}$

$$S_{1\mathrm{max}} = \frac{2 \cdot 2 \cdot 1.7 \cdot 1.5}{1.2} + \frac{2 \cdot 1\,500 \cdot 9.81 \cdot 0.05}{30 \cdot 1.2 \cdot 1.3 \cdot 1^2}\,\mathrm{m}^2\,,$$
$$S_{1\mathrm{min}} = \frac{2 \cdot 2 \cdot 1.7 \cdot 1.5}{1.2} + \frac{2 \cdot 1\,500 \cdot 9.81 \cdot 0.05}{30 \cdot 1.2 \cdot 1.3 \cdot 15^2}\,\mathrm{m}^2\,,$$
$$S_{1\mathrm{max}} = 39.9\,\mathrm{m}^2\,,$$
$$S_{1\mathrm{min}} = 8.7\,\mathrm{m}^2\,.$$

Since we cannot think of sails in the same way as fuel, where there is a precisely defined consumption for a defined distance, it is very difficult to even estimate the cost of their creation. They also wear out over time, undoubtedly sooner in the case of stormier weather.

We estimate the lifetime of the sail to be $s_\mathrm{z} = 200\,\mathrm{km}$, while the cost of its construction (including partial consideration of plant care and spinning mill operation) may be $E_\mathrm{v} = 1\,\mathrm{MJ}$.

At the same time, the following applies to the work done by the wind:

$$W = Fs\,,$$
$$s = \frac{W}{F} = \frac{1\,\mathrm{J}}{\frac{1}{2} C_1 S_1 \rho v^2}\,,$$
$$s_\mathrm{max} = \frac{2}{1.2 \cdot 39.9 \cdot 1.3 \cdot 1^2}\,\mathrm{m} = 3.2\,\mathrm{cm}\,,$$
$$s_\mathrm{min} = \frac{2}{1.2 \cdot 8.7 \cdot 1.3 \cdot 15^2}\,\mathrm{m} = 0.065\,\mathrm{cm}\,.$$

Thus, per 1 J of work done, there is (at most, i.e., in the most disadvantageous arrangement)

$$E = \frac{s_\mathrm{max}}{s_\mathrm{z}} E_\mathrm{v} = \frac{3.2 \cdot 10^{-2}\,\mathrm{m}}{200 \cdot 10^3\,\mathrm{m}} \cdot 10^6\,\mathrm{J} = 0.16\,\mathrm{J}\,.$$

From an energy point of view (in relation to the human cost), this is the case for land vessels, where approximately 1/6 J of work is done per 1 J.

It must be admitted, however, that they are entirely dependent on the vagaries of the weather, even though the eventual direction of travel could be influenced and changed to the desired cruising direction. Here, however, the problem of feasibility arises, since the limited space of the road does not allow such manoeuvres.

A possible modification to make the car ship handle the field terrain would, in most interpretations, involve changing the tires, which would contradict the reasoning done with the numerical values chosen above. It is also likely that even being able to climb, e.g., a field does not automatically imply permission to cross it (due to crop loss).

In short, this type of transport can be considered more energy efficient but not feasible.

Conclusion

Ranking of drive methods according to efficiency:

- auto-ship – $0.16\,\mathrm{J{\cdot}J^{-1}}$
- petrol – $8\,\mathrm{J{\cdot}J^{-1}}$

- electricity – $27\,\mathrm{J{\cdot}J^{-1}}$
- hydrogen – $29\,\mathrm{J{\cdot}J^{-1}}$
- steam engine
- horse (animal)

At the same time, however, it is clear that the first mean is only an ideal concept without practical use or at least actual application.

A certain defect of petrol is its dependence on fossil fuels, but this is not avoided by the electric car through lithium and by the steam engine through coal. From this point of view, only the hydrogen-powered car (and possibly animal power) can be considered a purely independent source of propulsion.

Petrol can be considered the most efficient way of powering a passenger car.

Corrector's comment

I agree with the written above to the full extent, except for the chapter on wind, which contains some factual inconsistencies and errors, e.g., as far as air resistance is concerned – in the case of joint motion with the air mass flow considered here, the environmental resistance will be applied minimally and much more complicated. A two hundred mile sail life is an extremely pessimistic estimate, given what sails on racing sailboats can withstand.

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