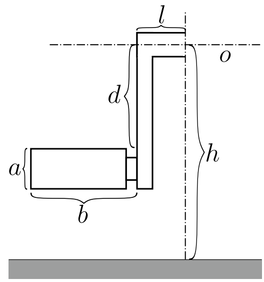
Problem I.2 ... turbocyclist 3 points



At what maximal velocity can a cyclist go through a left turn with a radius of $r = 7.5\,\mathrm{m}$, for him not to touch the ground with his pedal? The cyclist and his bicycle weigh $m = 80\,\mathrm{kg}$, their collective center of gravity is in the plane of symmetry of the bicycle. The pedal clique is at its lowest point the whole time. While the pedal itself is not turned either to the front or backwards, the position of the pedal in relation to the clique stays the same as depicted in the picture. Pedal is $a = 2.0\,\mathrm{cm}$ high and $b = 10\,\mathrm{cm}$ wide, the dimensions of the clique are $d = 15\,\mathrm{cm}$ and $l = 6.5\,\mathrm{cm}$, as can be seen from the picture. When the bicycle is in vertical position, the horizontal axis of the rotation of the clique around the frame of the bicycle o is $h = 30\,\mathrm{cm}$ above the ground. The vertical dashed line marks the vertical symmetry of the bicycle. The center of gravity also lies on this line, length l, therefore it is measured from the center of the bottom bracket. Friction is sufficient for the bicycle not to slip.

Terka was thinking whether she is going to crash on her bicycle (again).

The basis for solving the problem is a correct diagram. Let α denote the maximum angle by which the cyclist may deflect from the vertical position without the pedal scraping the road. The position of the pedal in this critical case is shown in figure 1.

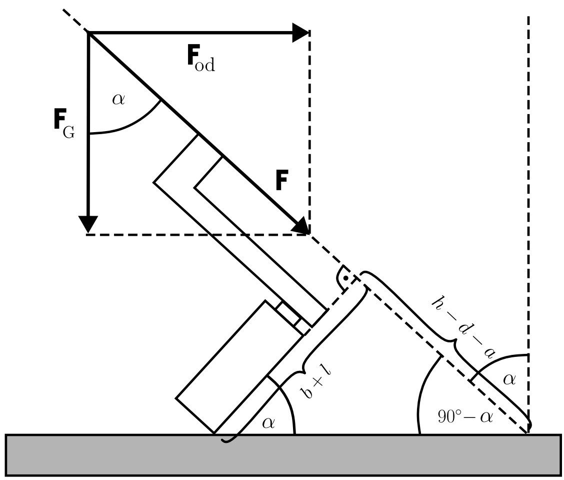


Figure 1: Diagram of the pedal position and forces acting on the cyclist and bicycle in the critical case.

It is clear that if contact with the ground occurs, it will be at the lowest outer edge of the pedal (the left bottom corner on the figure in the problem statement). It can be seen from the diagram, that the maximum angle of inclination α can be determined from the position of this edge. Let us consider a right triangle whose hypotenuse coincides with the surface of the road and whose legs are given by the plane of symmetry of the bicycle (the vertical axis in the figure in the problem statement) and the plane given by the underside of the pedal (the imaginary extension of the longer lower edge of the pedal).

The length of the first leg can be determined from the figure in the problem statement as "the height of the lower edge of the pedal above the ground", which, by simply subtracting the lengths, gives us the value $h - d - a$. The second leg on this figure represents the horizontal distance of the edge of the pedal from the plane of symmetry, and again, by simple addition, we obtain the length $b + l$. For the tangent of angle α, the following holds by definition

$$\tan \alpha = \frac{h - d - a}{b + l}\,. \tag{1}$$

During cornering, in the reference frame fixed to the road, two forces act on the cyclist and his bicycle: the gravitational force $\mathbf{F}_G$ and the centrifugal force $\mathbf{F}_{\mathrm{od}}$. Let us denote their resultant as $\mathbf{F} = \mathbf{F}_G + \mathbf{F}_{\mathrm{od}}$. The gravitational force acts downwards and its magnitude is

$$|\mathbf{F}_G| = F_G = mg\,, \tag{2}$$

where $m = 80\,\mathrm{kg}$ is the total mass of the cyclist and his bicycle and $g \doteq 9.81\,\mathrm{m{\cdot}s^{-2}}$ is the standard gravitational acceleration. If we consider a turn on flat ground, the centrifugal force $\mathbf{F}_{\mathrm{od}}$ acting on the cyclist will be perpendicular to the gravitational force at all times, and also perpendicular to the direction of motion. The magnitude of this force is defined as

$$|\mathbf{F}_{\mathrm{od}}| = F_{\mathrm{od}} = \frac{mv^2}{r}\,, \tag{3}$$

where v is the cyclist's speed and $r = 7.5\,\mathrm{m}$ the radius of curvature of the turn.

Let us now imagine the lean of the cyclist into the turn as a rotation about an axis passing through the wheel–road contact points (which is parallel to the direction of motion). If we assume that the above-mentioned forces will act at the center of gravity of the cyclist-bicycle system, which does not lie on this axis, a non-zero torque may act on this system about this axis.

If the torque were to deflect the cyclist counterclockwise (to the left), not only would the pedal touch the road, but at a certain point the bicycle wheels would lose contact with the road surface resulting in a fall. Conversely, if the torque were to cause clockwise deflection (to the right), the cyclist would have to start slowing down, else she would fly off the curved road after a certain time. It follows that if the cyclist is to go through a turn at a constant speed, the total torque about the axis passing through the points of contact of the wheels with the road must be zero. The magnitude of the torque M is given by

$$M = FR\sin\varphi$$

where F is the magnitude of the acting force, R is the distance of the axis of rotation from the point of application of the force (lever arm), and φ is the angle between the vector of the acting force and the lever arm vector. The resulting force with respect to the different directions $\mathbf{F}_G$ and $\mathbf{F}_{\mathrm{od}}$ will not be null, nor will the magnitude of the lever arm R, since the point of application of the forces lies off the considered axis. Therefore, it remains to ensure that the angle φ is zero; in other words, the line on which the resultant force vector $\mathbf{F}$ lies must intercept the axis of rotation. In our case, this means that $\mathbf{F}$ must lie in the plane of symmetry of the bicycle, as shown in diagram 1.

The resultant $\mathbf{F}$ is formed by the superposition of forces $\mathbf{F}_G$ and $\mathbf{F}_{\mathrm{od}}$, which are moreover perpendicular to each other. These three vectors form a right-angled triangle, and from the geometric ratios in the figure, it is clear that the remaining angles in the triangle will be α and $90^\circ - \alpha$. For the tangent of angle α according to the figure 1 and equations (2) and (3), the following holds

$$\tan\alpha = \frac{F_{\mathrm{od}}}{F_{\mathrm{G}}} = \frac{v^2}{rg}\,.$$

Substituting this expression into equation (1) yields

$$\frac{v^2}{rg} = \frac{h-d-a}{b+l}\,,$$

from which we express and calculate the maximum velocity v (positive). For parameter values $a = 2.0\,\mathrm{cm}$, $b = 10\,\mathrm{cm}$, $d = 15\,\mathrm{cm}$, $l = 6.5\,\mathrm{cm}$ and $h = 30\,\mathrm{cm}$, we obtain the final result

$$v = \sqrt{rg\left(\frac{h-d-a}{b+l}\right)} \doteq 7.6\,\mathrm{m{\cdot}s^{-1}} \approx 27\,\mathrm{km{\cdot}h^{-1}}\,.$$

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