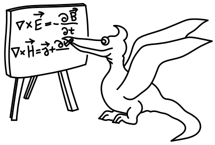

Serial: Plasma Diagnostics

Dear readers,
this part of the series on experimental methods in physics will focus on high-frequency (HF) plasma diagnostics. Plasma is a form of matter that naturally occurs in outer space, in stellar interiors, and in the atmosphere as auroras or lightning. In laboratory, we usually create plasma in sodium or xenon discharge lamps and in many technological devices used in semiconductor manufacturing, surface treatment, welding, cutting, and so on. Therefore, we will devote the first part of the series to defining plasma and the conditions plasma must satisfy.

Plasma

Definition of plasma

The most commonly used definition of plasma states: "Plasma is a quasi-neutral gas of charged and neutral particles exhibiting collective behavior."

As with most definitions, it does not tell us much, so let us break down the individual terms.

Quasi-neutrality means that any small volume of plasma containing a sufficiently large number of charged particles will have zero net charge. In other words, it contains as much charge from free electrons as from positively ionized atoms. However, the number of electrons and ionized atoms may differ because atoms may be multiply ionized.

The explanation of collective behavior is somewhat more complicated. In a classical gas such as air at atmospheric pressure, all particles are neutral, so we assume that between collisions no net electromagnetic force acts on them. In plasma, however, a non-negligible portion of particles is charged, so they interact via the Coulomb force and, more importantly, create their own electromagnetic field. If we introduce some external potential φ_0 into the plasma, the plasma responds in a way that suppresses this perturbation. The main reason for this behavior is the long-range nature of the Coulomb force.

The large mass difference between electrons and positively charged ions also influences this behavior. Although their charges have equal magnitude but opposite sign, the proton mass is roughly 2 000 times greater than that of an electron, so the action of an external force produces a much faster response in electrons. This allows us to assume that protons and other ions are stationary compared to electron motion. If an external perturbation in the plasma charge density appears, it is neutralized primarily by electrons. In a slightly positive region, electrons flow into that location; in a negative region, electrons are repelled away.

Plasma parameters

As mentioned in the introduction, the conditions under which plasma occurs vary greatly. Therefore, it is necessary to introduce quantities that can characterize plasma behavior. Five quantities should suffice: the electron density n_e, plasma temperature T, Debye length λ_D, plasma parameter N_D, and plasma frequency f_p.

The electron density n_e tells us how many electrons are present per unit volume. From the definition of plasma we know it is quasi-neutral, so the total electron charge must equal the charge of positive ions. For simplicity, if we neglect negative ions and multiply charged particles, then the ion density n_i equals the electron density, $n_\mathrm{i} = n = n_\mathrm{e}$. Often it is easier to measure the electron density than the ion density, because electrons have lower mass, higher mobility, and thus respond more quickly to external influences.

We may express the plasma temperature in kelvins or degrees Celsius, but in plasma physics it is also commonly given in electronvolts (eV). As we know from previous parts, one electronvolt corresponds to the energy gained by an electron accelerated through a potential difference of one volt, $1\,\mathrm{eV} \doteq 1.60 \cdot 10^{-19}\,\mathrm{J}$. Equating the thermal energy $k_\mathrm{B}T$, where $k_\mathrm{B} \doteq 1.38 \cdot 10^{-23}\,\mathrm{J{\cdot}K^{-1}}$ is the Boltzmann constant, with the energy gained by acceleration to $1\,\mathrm{eV}$ gives the conversion

$$1\,\mathrm{eV} \mathrel{\hat{=}} 11\,600\,\mathrm{K}\,.$$

Describing temperature in eV occurs more often in high-temperature plasma. For example, in ITER, currently the largest tokamak under construction, they aim to reach a temperature of $13\,\mathrm{keV}$, or 150 million kelvins.

Notably, plasma may have two different temperatures if it is not in thermodynamic equilibrium. Electrons and ions are very different particles, differing in mass and size, which leads to different collision frequencies and different temperatures. After sufficient time following the shutdown of the external ionization source, the system transitions to thermodynamic equilibrium and temperatures equalize. Two temperatures may also appear in the presence of an external magnetic field, when the plasma temperature differs parallel and perpendicular to the magnetic field lines. There may even be two groups of electrons with different temperatures.

The Debye length characterizes the distance over which a local disturbance of the charge density or potential influences the surrounding plasma. If a region has potential φ_0 relative to the plasma, charged particles are attracted to it and their fields shield the external potential. This effect is called Debye shielding. One can show that in a 1D case the potential depends on the distance x from the disturbance as

$$\varphi = \varphi_0 \exp\left(-\frac{|x|}{\lambda_\mathrm{D}}\right).$$

The decay is exponential and characterized by λ_D, the Debye length:

$$\lambda_\mathrm{D} = \sqrt{\frac{\varepsilon_0 k_\mathrm{B} T}{n_\mathrm{e} e^2}}\,.$$

It depends, apart from the vacuum permittivity $\varepsilon_0 \doteq 8.85 \cdot 10^{-12}\,\mathrm{F{\cdot}m^{-1}}$, elementary charge $e \doteq 1.60 \cdot 10^{-19}\,\mathrm{C}$, and k_B, on the plasma temperature T and electron density n_e.

If we take a sphere of radius λ_D and count how many charged particles it contains, we obtain the plasma parameter

$$N_\mathrm{D} = \frac{4}{3}\pi\lambda_\mathrm{D}^3 n_\mathrm{e}\,.$$

Besides spatial characterization, temporal behavior is also important. Electrons oscillate around stationary ions with the so-called plasma frequency

$$f_\mathrm{p} = \frac{1}{2\pi}\sqrt{\frac{n_\mathrm{e} e^2}{m_e \varepsilon_0}}\,.$$

If we attempt to send electromagnetic radiation into plasma, we find that the response depends on the chosen frequency. At sufficiently low frequencies, plasma behaves like a mirror and reflects the incident wave.

If we send a wave with frequency $f < f_\mathrm{p}$ into plasma, electrons can oscillate at the frequency of the incoming wave and create an electric field of opposite orientation; plasma therefore acts as a mirror. If the wave frequency exceeds f_p, the wave propagates inside the plasma.

The plasma frequency does not refer only to waves. Collisions in plasma, particularly with neutral particles, play an important role. According to the magnitude of the mean time between collisions τ, we classify plasma as collisional or collisionless.

Conditions for plasma existence

To determine whether a medium is truly plasma and satisfies the assumption of collective behavior, three fundamental conditions must hold:

$$\begin{aligned} \lambda_\mathrm{D} &\ll L\,, \\ N_\mathrm{D} &\gg 1\,, \\ f_\mathrm{p} &\gg 1/\tau\,. \end{aligned}$$

In other words, the characteristic plasma size L must be much larger than the Debye length to allow effective shielding. The number of particles N_D within the relevant volume must be sufficiently large to mediate shielding. Finally, the collision frequency $1/\tau$ must be smaller than the plasma frequency.

Types of plasma

Table 1 lists some plasma types together with approximate electron densities and temperatures. We see that the values differ by several orders of magnitude. Therefore, various classifications are introduced: collisional/collisionless, low/high-temperature, frozen/diffusing magnetic field, or even quantum or relativistic plasma.

Table 1: Order-of-magnitude values of temperature and electron density for various types of plasma.

type of plasma	$\frac{T_e}{\mathrm{K}}$	$\frac{n_\mathrm{e}}{\mathrm{m}^{-3}}$
intergalactic space	$10^4 - 10^8$	$10^0 - 10^2$
solar wind	$10^4 - 10^5$	$10^4 - 10^8$
solar corona	$10^5 - 10^7$	$10^{10} - 10^{14}$
stellar cores	$10^7 - 10^8$	$10^{28} - 10^{34}$
tokamaks	$10^7 - 10^8$	$10^{18} - 10^{22}$
glow discharges	10^4	$10^{14} - 10^{18}$

Description of plasma

How can we describe plasma? In general, there are two approaches: we either observe the behavior of the velocity and position of individual particles or study macroscopic quantities such as density or current (both electric and particle).

The first method is called kinetic or statistical description, and its main quantity is the velocity distribution function $f_\alpha(\mathbf{r}, \mathbf{v}, t)$. It specifies the velocity distribution $f_\alpha(\mathbf{v})$, i.e., what fraction of particles moves with velocity v, depending on species α (electrons or ions), position $\mathbf{r}$, and time t. The time evolution of plasma is described by the Boltzmann kinetic equation

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} f + \frac{\mathbf{F}}{m} \cdot \nabla_{\mathbf{v}} f = \left(\frac{\partial f}{\partial t}\right)_s ,$$

which includes, besides the partial time derivative, gradients in position $\mathbf{r}$, gradients in velocity $\mathbf{v}$, and the effect of external force $\mathbf{F}$. The term on the right-hand side is the collision term, which, as the name suggests, accounts for collisions. If the collision term is zero, we obtain the Vlasov equation.

Kinetic theory allows a very accurate description of plasma behavior but is computationally demanding, because it involves calculating the evolution of the distribution function in space. If we average the Boltzmann equation over suitably chosen quantities, we obtain the second plasma description, called the fluid or MHD (magnetohydrodynamic) description.

The MHD description consists of a set of differential equations such as the continuity equation, the fluid momentum equation, and the equation of state. To these we add Maxwell's equations, which describe electromagnetic fields. This method can describe macroscopic phenomena such as various drifts in plasma or plasma behavior in external electromagnetic fields.

Plasma diagnostics

Just like plasma itself, plasma diagnostic methods are diverse. In general we may encounter probe-based, high-frequency, particle, and optical diagnostic techniques. Moreover, different methods allow measurement of only certain quantities—probe diagnostics can determine plasma density, electron temperature, or their velocity distribution; high-frequency methods primarily measure electron density; and particle or optical methods can also provide information about plasma chemistry.

In addition to plasma parameters, experimental conditions also matter. Most academic and applied research works with plasma in a vacuum chamber, i.e., at low pressures. Besides chamber pressure, the type and flow rate of the gas being ionized also influence the plasma. Equally important is the method of plasma generation, e.g., by an electric field from cold or heated electrodes, by radiation (microwave or ultraviolet), or by induced electric fields.

Langmuir probe

With a bit of exaggeration, one of the most common plasma diagnostic methods is simply a piece of wire inserted into the plasma. This is the Langmuir probe, typically a tungsten wire a few millimeters long and tens to hundreds of micrometers in diameter. The probe voltage is varied with respect to another, much larger, so-called reference electrode, which may even be the metal wall of the vacuum chamber. By recording the current for each voltage, we obtain the current-voltage (probe) characteristic. From this, we can determine not only the electron density but also electron temperature or velocity distribution.

A concise description and function of the Langmuir probe were presented in the previous serial of FYKOS, 26th year, 5th series[1], which we recommend to interested readers.

High-frequency plasma diagnostics

High-frequency diagnostic methods are traditionally divided into free-space methods, where we study changes in radiation passing through plasma; resonator methods, where a resonator is partially filled with plasma; and waveguide methods, where plasma is placed inside a waveguide. Such diagnostics fall into the category of non-contact methods, because no electrodes are inserted into plasma. However, we may also feed a high-frequency signal, typically in the GHz range, into the plasma using an inserted antenna and study how the signal interacts with the plasma. If the antenna acts as a resonator, then by determining the shift of its resonant frequency f_r in plasma we can determine the relative permittivity ε_r and thus the electron density n_e.

The basic examples of high-frequency resonators are the Hairpin probe and Slot antenna, see Fig. 1.

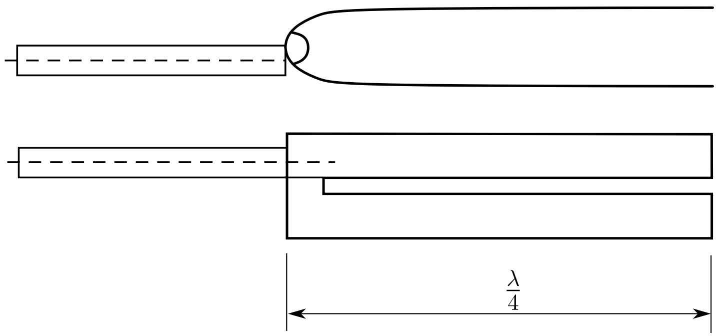

Figure 1: Quarter-wavelength resonators: Hairpin probe and Slot antenna.

Both probes behave as quarter-wave resonators, with resonant frequency

$$f_\mathrm{r} = \frac{c}{4L\sqrt{\varepsilon_\mathrm{r}}}\,.$$

L is the characteristic length of the probe, usually the length of the antenna; $\lambda = 4L$ is the wavelength of the electromagnetic wave fed to the probe via a coaxial cable; and ε_r is the relative permittivity of the medium surrounding the probe.

During measurement, we compare the resonant frequency of the probe without plasma f_0, where $\varepsilon_\mathrm{r} = 1$, with the resonant frequency in plasma f_r. Since the relative permittivity of plasma is slightly less than 1, the resonant frequency shifts upward.

As already mentioned in the section *Plasma conditions*, we must send a signal into plasma whose plasma frequency f exceeds the plasma frequency f_p, otherwise the wave reflects. The relation between relative permittivity and plasma frequency is

$$\varepsilon_\mathrm{r} = 1 - \frac{f_\mathrm{p}^2}{f^2}\,.$$

[1] `https://static.fykos.cz/problems/fykos/26/media/serial5.cs.pdf`

From the previous two relations, we derive plasma frequency in terms of the difference of the squared resonant frequencies with and without plasma:

$$f_\mathrm{p}^2 = f_\mathrm{r}^2 - f_0^2\,.$$

In the section *Plasma parameters* we stated the formula for plasma frequency. Other than known constants, it depends only on electron density, which we may therefore determine from the measured plasma frequency.

Let us return to the formula for relative permittivity $\varepsilon_\mathrm{r} = 1 - f_\mathrm{p}^2/f^2$ and to the relationship between the resonator frequency and the plasma frequency $f_\mathrm{r} > f_\mathrm{p}$. Since the denominator is greater than the numerator, the relative permittivity must be less than one. This is another special property of plasma, distinguishing it from other forms of matter, which have $\varepsilon_\mathrm{r} > 1$. This also affects the phase velocity of light in plasma $c' = c/\sqrt{\varepsilon_\mathrm{r}}$, which, due to $\varepsilon_\mathrm{r} < 1$, comes out greater than the speed of light in vacuum c. This does not contradict special relativity because the wave itself does not carry information; only modulations of the wave do, and these propagate at the group velocity, which is always less than c.

Curling probe

The Curling probe also belongs to high-frequency plasma diagnostics. It has the shape of a spiral obtained by "rolling up" a Slot antenna, hence the name Curling. The characteristic length L is now the length of the spiral, see Fig. 2. The change in geometry also requires a change in probe construction: the previous probes were made of electrically conductive wire directly connected to a coaxial cable, whereas the Curling probe consists of an electrically non-conductive circular substrate (quartz glass) coated with a thin layer of conductive metal (copper) about 0.1 mm thick in the shape of a spiral. The excitation is provided by an antenna inside the holder of the Curling probe.

The Langmuir probe, Hairpin probe, and Curling probe also differ in the spatial character of their measurements. The Langmuir probe is significantly smaller than the other probes, so it measures essentially local plasma parameters at its position. The other two probes are larger and average parameters over the region to which they are exposed. The Hairpin probe averages mainly over the plasma volume enclosed by the antenna, while the Curling probe averages over the planar region of plasma adjacent to it.

The main application of these probes is in so-called technological plasma, used for industrial production of thin films or semiconductor chips. However, if a thin film begins to deposit on a Hairpin or Slot probe, their characteristics—and thus their measurement properties—change. As we will show, the Curling probe can compensate for this effect through calibration.

The change in probe geometry and the addition of a dielectric substrate in the Curling probe unfortunately has one drawback: the simple formula $f_\mathrm{r} = c/(4L\sqrt{\varepsilon_\mathrm{r}})$ no longer applies. The complex geometry and dielectric material prevent the use of the previous method for determining electron density. The relationship between plasma resonant frequency f_r and relative permittivity ε_r must therefore be constructed experimentally through calibration.

Calibration

Calibration is performed using various materials with different relative permittivities in the microwave range. These materials are electrical insulators such as ceramics, glass, or plastics,

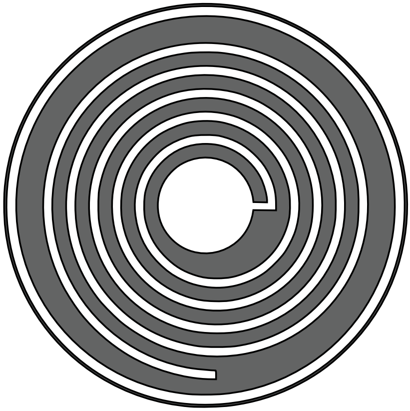

Figure 2: Curling probe; the dark area marks the conductive part of the probe. The spiral length L is determined by the uncoated (white) part of the spiral.

see Table 2. For selected materials and for measurement in vacuum (air), we can measure the dependence $f_\mathrm{r}(\varepsilon_\mathrm{r})$ of the form

$$f_\mathrm{r}(\varepsilon_\mathrm{r}) = \frac{c}{4L\beta\left(\frac{\varepsilon_\mathrm{r}+\alpha}{2}\right)^\gamma}\,,$$

including free parameters α, β, and γ. By fitting the measured data, we can extrapolate the dependence $f_\mathrm{r}(\varepsilon_\mathrm{r})$ into the range where ε_r is several tenths below one, i.e., into the plasma region. Inverting the expression into $\varepsilon_\mathrm{r}(f_\mathrm{r})$ and using $f_\mathrm{p}^2 = f_\mathrm{r}^2(1-\varepsilon_\mathrm{r}(f_\mathrm{r}))$ yields the electron density

$$n_\mathrm{e} = (2\pi)^2\frac{m_e\varepsilon_0}{e^2}f_\mathrm{r}^2\left(1-\varepsilon_\mathrm{r}(f_\mathrm{r})\right).$$

Table 2: Relative permittivity of selected materials.

material	$\frac{\varepsilon_\mathrm{r}}{1}$
vacuum (air)	1.0
PTFE	2.2
ROGERS	3.7
MACOR	5.8

Analysis of resonance

Important parameters of a resonance are f_FWHM and the Q-factor of the peak. FWHM ("Full width at half maximum") denotes the frequency span f_FWHM at half the height of the resonant peak and, together with f_r, determines the resonance quality, see Fig. 3. The Q-factor is defined as

$$Q = \frac{f_\mathrm{r}}{f_\mathrm{FWHM}}\,.$$

Another interpretation follows from the equivalent definition

$$Q = 2\pi \frac{E_{\text{in}}}{E_{\text{cycle}}} \,,$$

where E_{in} is the energy stored in the resonator and E_{cycle} is the energy dissipated during one cycle. Thus the Q-factor characterizes energy loss per cycle. Simply put, higher Q means higher resonance quality and therefore higher measurement quality.

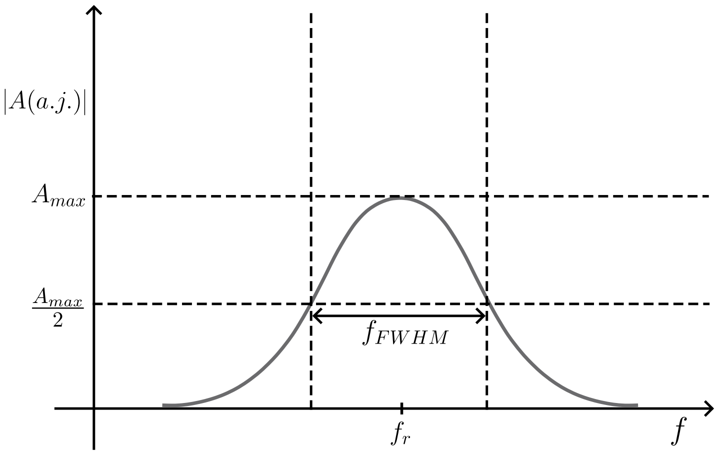

Figure 3: Determining f_{FWHM} and f_{r} from the resonance peak.

In general, high-frequency techniques can be modeled as driven, damped resonators. The differential equation

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = F_0 \exp\left(i\omega t\right) ,$$

contains a quantity proportional to the amplitude $x(t)$, the damping factor γ, the natural frequency ω_0, the driving amplitude F_0, and the driving frequency ω. By varying ω, we observe the system's response. Substituting a solution of the form $x(t) = A \exp\left(i\omega t\right)$ gives the amplitude magnitude $|A|$ that we can measure for given F_0, ω_0, γ, and variable ω:

$$|A(\omega)| = \frac{F_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}} \,.$$

Assuming that we are close to the natural frequency $\omega \approx \omega_0$, we may use the approximation[2]

$$\omega_0^2 - \omega^2 = (\omega_0 + \omega)(\omega_0 - \omega) \approx 2\omega_0(\omega_0 - \omega) \,.$$

Thus we obtain

$$|A(\omega)| \approx \frac{F_0}{\sqrt{4\omega_0^2(\omega_0 - \omega)^2 + (2\gamma\omega_0)^2}}$$

[2] Although this approximation seems rough, for microwave frequencies $\approx$ 1 GHz we fit regions of width 10 MHz, i.e., roughly 1 % of the resonant frequency.

which corresponds to the Cauchy distribution.

In HF diagnostics, we examine how much of the input power dissipate into the plasma near the resonant frequency $2\pi f_\mathrm{r} = \omega_0$. The attenuation is related to the Q-factor, $Q = \omega_0/(2\gamma)$. The Cauchy distribution can be rewritten in terms of the resonant frequency and the Q-factor:

$$A(f) = \frac{A_0}{2\pi} \frac{1}{f_\mathrm{r}\sqrt{(f_\mathrm{r} - f)^2 + \left(\frac{f_\mathrm{r}}{2Q}\right)^2}} \,.$$

Finally, we adjust for the measured quantity. The amplitude A may be compared to electric voltage U, but we usually measure reflected power $P \propto U^2$. In normalized form, we obtain

$$P(f) = P_0 \frac{1}{1 + \left(\frac{2Q(f_\mathrm{r} - f)}{f_\mathrm{r}}\right)^2} \,.$$

By fitting the measured dependence $P(f)$ with this function, we can determine the key peak parameters Q, f_r, and P_0. In the simplest case, these parameters may be read manually from the graph, though with lower accuracy.

When measuring power P, in addition to the unit W, we may also encounter the unit dBm. This logarithmic unit is used when the measured power spans several orders of magnitude during the experiment. The conversion between P_dBm and P_W is

$$P_\mathrm{dBm} = 10 \log_{10}\left(\frac{P_\mathrm{W}}{1\,\mathrm{mW}}\right) \,.$$

Conclusion

In this part of the serial, we outlined only a small portion of plasma physics. For interested readers, we recommend the previous series focused on fusion plasma[3]. We hope this series will inspire you to explore other areas of plasma physics, such as space plasma, plasma waves, or plasma chemistry. Worth special mention are computer simulations of plasma, which today—alongside theory and experimental techniques—form an essential part of advancing our understanding of plasma behavior.

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[3] `https://fykos.cz/archive/serial/26`