Problem V.P ... Dodo doesn't want to die 9 points

Which isotope is the most dangerous in terms of nuclear power plant accidents? Consider the amount that could be released during an accident, the likelihood of a leakage, the spreading of specific isotopes and their impact on the human body.

Dodo came to Jarda's for a beer and was afraid he wouldn't manage to finish the keg in time.

The structure of the solution is taken from our contestant Alena Mouchová.

Introduction

Radioactive isotopes gradually decay into more stable isotopes, most commonly by emitting *alpha*, *beta*, or *gamma* radiation.

The activity of a sample A is given by the equation

$$A \ = \lambda N = \frac{N \ln 2}{T_{1/2}}\,,$$

where $\lambda = \ln(2)/T_{1/2}$ is the decay constant and N is the number of atoms. The unit of activity is the becquerel (Bq), or equivalently s^{-1}. We calculate the number of atoms in the material as

$$N = nN_\mathrm{A} = \frac{m}{M} N_\mathrm{A}\,, \tag{1}$$

where n is the amount of substance, m is the mass of the sample, M is the molar mass, and N_A is the Avogadro constant. Furthermore, we can calculate the absorbed dose D as

$$D = \frac{E}{m_\mathrm{t}}\,,$$

where E is the absorbed energy and m_t is the mass of the material. The unit of the absorbed dose is the gray (Gy), or equivalently $\mathrm{J{\cdot}kg^{-1}}$.

The effects of radioactive radiation on a living organism can be stochastic or deterministic. Deterministic effects manifest within a short time (minutes to hours) after irradiation, and their severity increases with the received dose. These are the consequence of damage to or direct death of cells—nausea, vomiting, radiation burns, and others. However, some deterministic effects may not manifest until years later. Deterministic effects only appear above a certain threshold dose, because at low doses, the human body can usually cope with the damage. In contrast, stochastic effects manifest after a delay of years or decades, and the probability of their occurrence increases with the received dose. However, their severity is not linked to the size of the dose at all. Most commonly, this involves some form of cancer as a result of damage to genetic information.

Since different types of radiation cause different degrees of damage to living tissue, the equivalent dose is defined to account for the effects of irradiation in the human body

$$H = D w_\mathrm{r}\,,$$

where w_r are the radiation weighting factors for different types of radiation (e.g., 1 for electrons and photons, 20 for alpha particles). The equivalent dose is measured in sieverts (Sv), or equivalently $\mathrm{J{\cdot}kg^{-1}}$. Also, different tissues have different sensitivities to irradiation concerning stochastic effects. The effective dose E in the human body is calculated as the weighted sum of equivalent doses in various tissues and organs

$$E = \sum H w_{\mathrm{t}} \,,$$

where w_{t} are the tissue weighting factors. The unit of the effective dose is also the sievert, so one must always pay attention to which quantity given in sieverts is being discussed. The equivalent and effective doses are intended for assessing the stochastic effects of irradiation. Deterministic effects of irradiation depend on the absorbed dose.

The values of w_{r} and w_{t} are standardized according to the ICRP (International Commission on Radiological Protection), available on the SÚJB website[1].

A commonly used unit of energy in nuclear physics is the electronvolt (eV) and its multiples (keV, MeV). We have $1\,\mathrm{eV} = 1.602 \cdot 10^{-19}\,\mathrm{J}$.

Uranium 235

Uranium 235 is the fissile isotope in nuclear power plants. Uranium 235 is rare in nature (0.72 %) in natural uranium), and therefore its percentage in the fuel relative to uranium 238 needs to be increased, i.e., enriched. It primarily undergoes alpha decay on its own, but it spontaneously fissions with a probability of $7 \cdot 10^{-9}\,\%$.[2] During spontaneous fission, the heavy nucleus splits into two comparably heavy nuclei (proton number in the tens), and simultaneously, one or more fast neutrons are often released (energy on the order of MeV). At the same time, uranium 235 also has a high cross-section* for the fission reaction with thermal neutrons (energy $\lessapprox 0.5\,\mathrm{eV}$), where interaction with a thermal neutron causes the spontaneous fission of the uranium 235 nucleus.

Therefore, moderators (water, heavy water, graphite, …) are used in nuclear reactors to slow down the fast neutrons. Pressurized water reactors in the Czech nuclear power plants Dukovany and Temelín, moderated by water, use fuel enriched to 1.3–3.8 % uranium 235.[3] However, reactors moderated by much more efficient heavy water (D_2O) can even operate with natural uranium, but this is offset by the high cost of heavy water. The fission of uranium 235 produces secondary radioactive products such as iodine 131, cesium 137, and strontium 90.

Energy released during fission The fission of one U-235 atom releases approximately 200 MeV of energy. But how much is released during the fission of 1 kilogram? First, we must calculate the number of U-235 atoms in 1 kg. We do this using equation (1)

$$N_U = \frac{1\,000\,\mathrm{g}}{235\,\mathrm{g{\cdot}mol^{-1}}} \cdot 6.022 \cdot 10^{23}\,\mathrm{mol^{-1}} = 2.56 \cdot 10^{24} \,.$$

Then we need to find the total energy released during the fission of such an amount of atoms. If the fission of one U-235 atom produces an energy of 200 MeV, the total energy released by the fission of 1 kilogram of uranium will be

$$E_U = 2.56 \cdot 10^{24} \cdot 200\,\mathrm{MeV} \cdot 1.602 \cdot 10^{-13}\,\mathrm{J{\cdot}MeV^{-1}} = 8.2 \cdot 10^{13}\,\mathrm{J} \,.$$

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[1] `[https://sujb.gov.cz/fileadmin/sujb/docs/radiacni-ochrana/ICRP103_dokument.pdf](https://sujb.gov.cz/fileadmin/sujb/docs/radiacni-ochrana/ICRP103_dokument.pdf)`

[2] `[https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html](https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html)`

[3] `[https://www.cez.cz/nextcez/cs/o-cez/vyrobni-zdroje/jaderna-energetika/jaderna-energetika-v-ceske-republice/ete/technologie-a-zabezpeceni-1](https://www.cez.cz/nextcez/cs/o-cez/vyrobni-zdroje/jaderna-energetika/jaderna-energetika-v-ceske-republice/ete/technologie-a-zabezpeceni-1)`

This is an absurdly large amount of energy that would cause a massive explosion if released all at once. Fortunately, in nuclear reactors, fission energy is released stably and continuously, and the design of nuclear reactors usually includes negative feedback mechanisms intended to prevent an uncontrolled fission reaction.

Half-life and radioactivity As we already know, there are $2.56 \cdot 10^{24}$ atoms in 1 kilogram. The half-life of the U-235 isotope is $T_{1/2}^{U} = 704$ million years. The total activity of one kilogram from equation (1) will therefore be

$$A_U = \frac{2.56 \cdot 10^{24} \cdot \ln 2}{2.2 \cdot 10^{16}\,\mathrm{s}} \approx 8 \cdot 10^{7}\,\mathrm{Bq}\,.$$

This calculation shows that uranium 235, even though it has a very long half-life, can still release a significant amount of radiation in the event of an accident.

Health effects If uranium is inhaled or ingested, it can lead to serious health problems, mainly due to alpha radiation and chemical toxicity (uranium is a heavy metal). Although alpha radiation does not have a long range and is stopped by a few centimeters of air, a sheet of paper, or a layer of dead cells on the surface of human skin, this is precisely where its danger lies upon ingestion or inhalation of an alpha emitter. While alpha radiation does no harm in the layer of dead cells, inside the body, on the contrary, all the energy of an alpha particle on the order of MeV is released into living cells. Long-term exposure to alpha radiation can cause lung cancer (for example, when inhaling high concentrations of radon) or other serious diseases.

Plutonium 239

Plutonium 239 is an artificially prepared radioactive isotope that originates from uranium 238 after neutron capture and subsequent double beta decay. It is a significant alpha emitter with high toxicity. This isotope is of fundamental importance in both nuclear energy and military applications – it is one of the basic components of nuclear weapons and can simultaneously serve as fuel for nuclear reactors.

Half-life and radioactivity The half-life of Pu-239 is approximately $T_{1/2}^{Pu} = 24$–100 years. This means that its radioactive radiation persists for thousands of years. To calculate the activity of one kilogram of plutonium, we first calculate the number of atoms in 1 kilogram as

$$N_{Pu} = \frac{1\,000\,\mathrm{g}}{239\,\mathrm{g{\cdot}mol^{-1}}} \cdot 6.022 \cdot 10^{23}\,\mathrm{mol}^{-1} = 2.52 \cdot 10^{24}\,,$$

and by substituting into formula (1), we obtain the activity

$$A_{Pu} = \frac{2.52 \cdot 10^{24} \ln 2}{7.6 \cdot 10^{11}\,\mathrm{s}} \approx 2.3 \cdot 10^{12}\,\mathrm{Bq}\,.$$

Compared to $A_U \approx 8 \cdot 10^{7}$ Bq, we see an activity orders of magnitude higher, caused by the shorter half-life of plutonium.

Health effects Plutonium 239 is extremely toxic, especially upon inhalation or ingestion. Although alpha particles do not penetrate the skin, inside the organism they can cause serious damage to lung tissue, bone marrow, and other organs. Exposure leads to an increased risk of cancer, genetic mutations, and other serious diseases.

Due to its long half-life, plutonium 239 poses a long-term threat to the environment and human health. It remains dangerous in contaminated areas for centuries.

Cesium 137

Cesium 137 is a fission product created during the nuclear fission of uranium 235 or plutonium 239. Due to its chemical properties, it behaves similarly to potassium in the organism – it is easily absorbed and deposited in soft tissues, especially muscles. This makes it a significant threat to human health.

The half-life of cesium is $T_{1/2}^{Cs} = 30.2\,\mathrm{years}$, which, after conversion, corresponds to an activity of $A_{Cs} = 3.2 \cdot 10^{15}\,\mathrm{Bq}$, thus by far the highest of the isotopes discussed so far.

Health effects Cesium 137 is dangerous primarily due to the combination of penetrating gamma radiation and biological availability. After internal entry into the organism, it is distributed similarly to potassium and can damage soft tissues and bone marrow. Long-term exposure increases the risk of cancer, especially leukemia and bone marrow tumors.

The biological half-life in the human body is approximately $T_b \approx 110\,\mathrm{days}$, which means that the isotope remains in the body for a relatively long time. In the environment, cesium 137 is highly mobile—it easily contaminates water, soil, and the food chain, and given its 30-year half-life, it remains a significant threat to ecosystems for decades.

Iodine 131

Iodine 131 is one of the main fission products of uranium 235. It stands out for its biological activity because it selectively accumulates in the thyroid gland, where it can seriously damage tissue and increase the risk of developing cancer. It is particularly dangerous shortly after a nuclear accident because it is quickly released into the environment and, due to its short half-life, reaches high activity.

The half-life of iodine is only $T_{1/2}^{I} = 8\,\mathrm{days}$, which, after conversion, corresponds to a very high activity of $A_I = 5 \cdot 10^{18}\,\mathrm{Bq}$.

Health effects Iodine 131 concentrates in the thyroid gland, where its beta and gamma radiation damages cells and increases the risk of developing thyroid cancer. It poses the greatest threat to children and adolescents, in whom iodine metabolism is most intensive.

The danger is acute especially in the first days after an accident, when the activity of iodine 131 is highest. As a preventive measure, tablets of stable potassium iodide are therefore administered; they saturate the thyroid gland with inactive iodine and block the accumulation of the radioactive isotope. These tablets must be taken as soon as possible, ideally within 6 hours of exposure.

Thanks to its short half-life, the danger of iodine 131 decreases relatively quickly. After several months from the accident, it practically no longer poses a radiological risk.

Summary

Isotope	$A\,[\mathrm{Bq}]$	Health	Time risk	Radiation type
U-235	$8 \cdot 10^{7}$	cancer, genetic mutations	very long-term	α
Pu-239	$2 \cdot 10^{12}$	cancer, genetic mutations	long-term	α
Cs-137	$3 \cdot 10^{15}$	bone marrow damage	long-term	β, γ
I-131	$5 \cdot 10^{18}$	thyroid cancer	short-term	β, γ

Table 1: Summary of the properties of individual radionuclides.

Other radionuclides

The radionuclides mentioned above are, of course, not all that could be released during a nuclear accident, but they represent a selection of the most dangerous ones to humans. Similarly, we could also discuss, for example, the radioactive isotope strontium 90, cobalt-60, or americium 241.

Conclusion

Based on the performed calculations and research, it can be stated that the most dangerous isotopes released during a nuclear accident include primarily iodine 131 and cesium 137. Iodine 131 is a highly active radionuclide with a short half-life of eight days, which means it poses an acute risk especially in the first days after an accident. Its ability to selectively accumulate in the thyroid gland makes it a serious health threat, particularly for children and adolescents, whose iodine metabolism is more intensive. Fortunately, its danger decreases relatively quickly, and after several months from the accident, it practically no longer poses a significant risk.

Cesium 137, on the other hand, has a half-life of approximately thirty years, and its danger lies primarily in its long-term persistence in the environment. This isotope easily binds in soil and water, penetrates the food chain, and due to its chemical similarity to potassium, is deposited in soft tissues, especially in muscles. Consequently, it can cause damage for decades and thus poses a long-term environmental and health threat.

Plutonium 239 and uranium 235 are not as immediately dangerous because their radiation is predominantly alpha, which does not penetrate the skin. Nevertheless, both elements are highly toxic upon inhalation or ingestion, when they enter the organism directly and can cause severe health complications, including lung and bone marrow cancer. Their very long half-lives mean that they represent a permanent burden on the environment and human health.

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