Problem V.3 ... they got off on the orbit 5 points; průměr 3,82; řešilo 51 studentů

Tomáš boarded a train, sat in one of the carriages, and decided to take a nap. When he woke up, he found out he was alone in the carriage, and the carriage, along with him, was orbiting the Earth at an altitude of $h = 400\,\mathrm{km}$ above the planet's surface in a state of weightlessness. The carriage was oriented perpendicular to both the orbit of its center of mass and the radial direction.

Tomáš was excited because he realized he could take advantage of the inhomogeneity of the Earth's gravitational field to measure the length of the carriage. He took two $1\,\mathrm{kg}$ calibration weights he always carried around for such occasions out of his bag and placed them on the opposite ends of the carriage. He placed a laser telemeter between them and measured their relative distance. He then started a stopwatch. After $t = 60\,\mathrm{s}$, he measured their distance again—it has since changed by $\Delta l = 4\,\mathrm{cm}$. What is the length of the carriage L he measured? Assume the force acting on the weights was constant. Earth's mass is equal to $M = 5.97 \cdot 10^{24}\,\mathrm{kg}$ and the mean radius of the Earth is equal to $R = 6\,371\,\mathrm{km}$.

Tomáš has not traveled by train for a long time.

main circular orbits with a radius of $R+h$. These orbits intersect at two opposite points. When Tomáš started the measurement, the weights were at their farthest distance from each other. A cross-section of their trajectories is shown in figure 1.

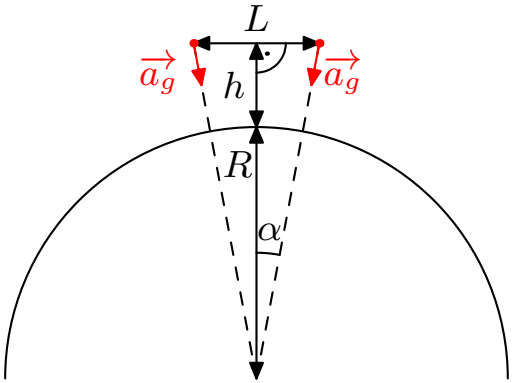

Figure 1: A sketch of the situation in a modified scale, with the weights orbiting the Earth in a direction out of the plane of the image.

In the direction of their mutual connection, each calibration weight moves with an acceleration

$$a_\mathrm{g} \sin\alpha = \frac{MG}{(R+h)^2} \frac{L}{2(R+h)}\,.$$

From the distance traveled by one of the calibration weights in the time interval Δt, we

calculate the length of the wagon as

$$\begin{aligned}\frac{\Delta l}{2} &= \frac{1}{2}at^2\,,\\ \Delta l &= \frac{MG}{(R+h)^3}\frac{L}{2}t^2\,,\\ L &= 2\frac{(R+h)^3\Delta l}{MGt^2} = 17.3\,\mathrm{m}\,.\end{aligned}$$

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