Problem V.2 ... waterlift

3 points; průměr 2,85; řešilo 54 studentů

We fill a hose with water—one end is submerged in a water reservoir, while the other end is sealed airtight. We then start pulling the hose vertically upward. This water has a temperature of 20 °C. How much would the height change if we used water at 90 °C instead, i.e., what is the difference between these two heights?

Hint: *Consider how strong the vacuum will be at the sealed end.*

Jarda was thinking about an experimental problem.

The water will remain in the hose because of the atmospheric pressure p_a acting on the reservoir surface, counteracting the weight of the water. We can analyze the situation at the reservoir surface, where the pressure balance between atmospheric and hydrostatic pressure must hold

$$p_a = H\rho g\,,$$

where H is the maximum height of the water column that can be drawn above the surface, ρ is the density of water, and g is the gravitational acceleration. At first glance, this equation does not contain anything that explicitly depends on temperature, suggesting that the maximum height remains constant.

But what would happen if we raised the sealed end above H? The water would not rise any higher; it would create a vacuum above its surface. This vacuum is not going to be be perfect, as water molecules will evaporate from the water surface inside the hose, forming saturated water vapor in that space. This vapor exerts a pressure p_{vap}; this pressure has to be added to the hydrostatic pressure at the reservoir surface. Moreover, the pressure of saturated water vapor is temperature-dependent, meaning that the height indeed starts to depend on temperature. This yields

$$p_a = h\rho g + p_{vap} \quad \Rightarrow \quad h = \frac{p_a - p_{vap}}{\rho g}\,.$$

Though there is one more temperature-dependent quantity in this equation: the density of water. We can find the necessary values online. At $T = 20\,^\circ\mathrm{C}$, the density of water equals $\rho_{20\,^\circ\mathrm{C}} \doteq 998\,\mathrm{kg\cdot m^{-3}}$, while at $90\,^\circ\mathrm{C}$, it equals $\rho_{90\,^\circ\mathrm{C}} \doteq 965\,\mathrm{kg\cdot m^{-3}}$.[1] Similarly, we must check the tabled values for the vapor pressure of water $p_{vap,90\,^\circ\mathrm{C}} \doteq 70\,\mathrm{kPa}$ at the higher temperature and $p_{vap,20\,^\circ\mathrm{C}} \doteq 2.3\,\mathrm{kPa}$.[2] We see that at room temperature, this vapor pressure is negligible compared to the atmospheric pressure, but it is certainly not the case at higher temperatures.

[1] https://www.engineeringtoolbox.com/water-density-specific-weight-d_595.html

[2] https://www.omnicalculator.com/chemistry/vapour-pressure-of-water

Substituting the numbers

$$h_{20\,^\circ\mathrm{C}} = \frac{p_\mathrm{a} - p_{\mathrm{vap},20\,^\circ\mathrm{C}}}{\rho_{20\,^\circ\mathrm{C}} g} \doteq 10.1\,\mathrm{m}\,,$$
$$h_{90\,^\circ\mathrm{C}} = \frac{p_\mathrm{a} - p_{\mathrm{vap},90\,^\circ\mathrm{C}}}{\rho_{90\,^\circ\mathrm{C}} g} \doteq 3.3\,\mathrm{m}\,,$$
$$\Delta h = h_{20\,^\circ\mathrm{C}} - h_{90\,^\circ\mathrm{C}} = 6.8\,\mathrm{m}\,.$$

At 20 °C, we can draw water about 6.8 m higher than at 90 °C, which is approximately three times as high.

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