Problem V.1 ... square?

3 points; průměr 2,49; řešilo 51 studentů

Jindra owns a sheet of paper with a square hole cut out in the middle. The hole has a side length $a = 3\,\mathrm{mm}$. It is a beautiful sunny day, so Jindra takes this sheet of paper outside to project his square onto the sidewalk. First, he placed the paper about 2 mm above the sunlit surface. What shape does the light spot on the wall caused by the square hole in the paper have? Jindra then moved his favorite piece of paper to a distance of about 1.5 m above the sidewalk. What shape does the light spot on the sidewalk have now? Explain why the shadow behaves this way. *Jindra solved the squaring of a circle.*

of $\sim 2\,\mathrm{mm}$, the bright spot takes the shape of a square. However, after the paper is moved to a much greater distance of $\sim 1.5\,\mathrm{m}$, the bright spot takes the shape of a circle. In this case, the hole functions similarly to a pinhole camera, projecting an image of the Sun onto the pavement.

The key lies in comparing the angular sizes of the hole and the Sun as seen from the pavement. The Sun has an angular diameter of approximately $\theta = 0.5^\circ$. At a distance of $d = 2\,\mathrm{mm}$, the square hole has an angular diameter of approximately

$$\alpha \approx 2\arcsin\left(\frac{a}{2d}\right) = 1.7\,\mathrm{rad} = 97^\circ\,,$$

which is much larger than the Sun's angular diameter θ. Thus, when viewed from the ground through the square hole, the Sun acts as a point source, and the bright spot retains a square shape.

On the other hand, at a distance $d = 1.5\,\mathrm{m}$, the square hole has an angular diameter

$$\beta = \frac{a}{d} = 0.002\,\mathrm{rad} = 0.11^\circ\,,$$

where we used the small-angle approximation $\sin(\beta) = \beta$. This angular diameter is now smaller than the Sun's angular diameter θ. In this case, the square hole behaves like a pinhole camera, projecting a circular image of the Sun.

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