

Problem IV.E ... antidune

12 points

Find all-purpose flour and poppy seeds in the kitchen. Verify whether their angle of repose depends on the height of the pile formed by pouring. Additionally, measure the dependence of the angle of repose on the mass concentration of the mixture of plain flour and poppy seeds.

Karel was thinking of adding salt and Tomáš of makówki.

Theoretical Background

When pouring granular material onto one spot, it does not spread evenly across the surface or remain where it lands. Instead, it forms a cone-shaped heap. The angle between the cone's surface and its base is called the *angle of repose*. From a microscopic point of view, we can consider the forces acting on individual particles that are responsible for the overall arrangement. If a particle does not move, the acting forces are in equilibrium. A particle on the slope is pulled downwards by gravity, but as it rolls, it is hindered by interactions with other particles, which on the macroscopic scale manifest as friction.

Using the coefficient of friction, the angle of repose of various materials can be estimated in a first approximation. Consider a model of sliding with friction on an inclined plane, illustrated in Figure 1. In the limiting case, where the component \vec{F} of the gravitational force \vec{F}_G along the slope equals the maximum frictional force \vec{F}_t , we have

$$\vec{F} = -\vec{F}_t.$$

From the geometry of the problem,

$$mg \sin \theta = mgf \cos \theta,$$

where m is the particle mass, g the gravitational acceleration, θ the limiting angle (angle of repose), and f the static friction coefficient. Thus,

$$\tan \theta = f.$$

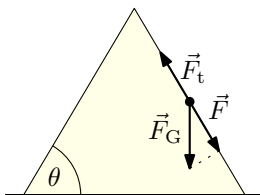


Figure 1: Forces acting on a small particle on the cone surface.

The angle of repose is also influenced by additional forces that may act between particles (e.g., adhesion, cohesion, and capillary effects when moist), but these are neglected given our measurement precision. The pouring rate can also affect the angle—particles poured too quickly tend to bounce, so material should be poured slowly during measurement.

Because the “shape” of the cone slope does not depend on its height, and therefore the macroscopic friction affecting falling particles remains constant, the angle of repose should also

be height-independent. When changing the mass fraction of poppy seeds and flour, we expect the angle of repose to vary within the range of the pure components.

The mass concentration c of a substance with mass m_1 in a mixture with mass m_2 is defined as

$$c = \frac{m_1}{m_1 + m_2}. \quad (1)$$

Measurement

The experimental setup is shown in Figure 2. The granular materials (fine flour, poppy seeds, and their mixtures) were poured slowly through a funnel onto one spot. The resulting cone dimensions were measured using two rulers: one fixed horizontally with a sliding triangle for reading the radius r , and another fixed vertically for measuring the cone height h . The angle of repose was then calculated using

$$\theta = \arctan \frac{h}{r}. \quad (2)$$

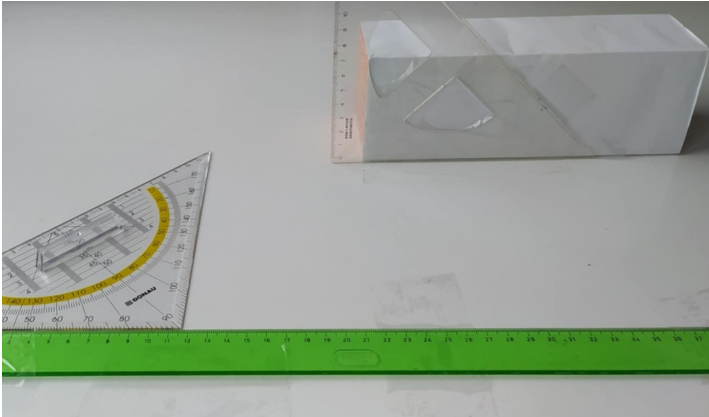


Figure 2: Experimental setup used.

The ruler accuracy was estimated as half the smallest scale division (0.5 mm), but additional uncertainty arises since the vertical ruler cannot be placed directly against the cone, and because the material does not always fall on exactly the same spot. Thus, the uncertainties in both σ_r and σ_h were estimated as 3 mm. Kitchen scales with uncertainty $\sigma_m = 1$ g were used to measure the masses of poppy seeds and flour for determining the mass fraction, calculated via Eq. (1).

Uncertainties of derived quantities were determined using standard propagation of uncertainty. The uncertainty σ_c in the mass concentration is given by

$$\sigma_c = \sqrt{\left(\frac{m_2}{(m_1 + m_2)^2} \sigma_{m_1}\right)^2 + \left(\frac{m_1}{(m_1 + m_2)^2} \sigma_{m_2}\right)^2}, \quad (3)$$

where m_1 and m_2 are the masses of poppy seeds and flour in the mixture, and σ_{m_1} , σ_{m_2} are their uncertainties. Since flour mass was kept constant at 100 g, the equation simplifies for subsequent measurements.

The measured cone parameters were used to compute the angle of repose θ from Eq. [eqrefR38S4U7](#) and its uncertainty σ_θ was obtained using a modified form of Eq. (3):

$$\sigma_\theta = \sqrt{\left(\frac{r}{h^2 + r^2}\sigma_h\right)^2 + \left(\frac{-h}{h^2 + r^2}\sigma_r\right)^2}. \quad (4)$$

Measured values are listed in Tables 1, 2, and 3.

Theoretical

Table 1: Measured values h, r, θ for flour.

$\frac{h}{\text{mm}}$	$\frac{r}{\text{mm}}$	$\frac{\theta}{^\circ}$
20	18	48 ± 9
28	27	46 ± 6
33	35	43 ± 5
36	34	47 ± 5
49	42	49 ± 4
53	59	42 ± 3
61	70	41 ± 3
75	79	44 ± 3
86	87	45 ± 2
91	99	43 ± 2
104	110	43 ± 2
112	145	38 ± 2

Table 2: Measured values h, r, θ for poppy seeds.

$\frac{h}{\text{mm}}$	$\frac{r}{\text{mm}}$	$\frac{\theta}{^\circ}$
26	46	30 ± 4
32	58	29 ± 3
36	60	31 ± 3
39	69	30 ± 3
41	71	30 ± 3
43	74	30 ± 2
45	73	32 ± 2
48	76	32 ± 2
50	79	32 ± 2
51	81	32 ± 2

Table 3: Mass of poppy seeds m_1 and their mass concentration c in a mixture with flour of mass 100 g; measured values of h and r ; dependence of the angle of repose θ on concentration c .

$\frac{m_1}{\text{g}}$	$\frac{c}{\%}$	$\frac{h}{\text{mm}}$	$\frac{r}{\text{mm}}$	$\frac{\theta}{^\circ}$
21	17.4 ± 0.7	44	43	46 ± 3
41	29.1 ± 0.6	49	46	47 ± 3
62	38.3 ± 0.5	46	52	41 ± 3
84	45.7 ± 0.4	51	53	44 ± 3
104	51.0 ± 0.4	46	52	41 ± 3
124	55.4 ± 0.4	54	62	41 ± 2
145	59.2 ± 0.3	38	52	36 ± 3
167	62.6 ± 0.3	42	49	41 ± 3
187	65.2 ± 0.3	51	55	43 ± 3
200	66.7 ± 0.3	47	57	40 ± 3

Subsequently, these data were fitted with a straight line of the form $y = ax + b$ in Python 3 using the ODR function from the `scipy` library. During the fitting procedure, uncertainties of

both the dependent and independent variables were taken into account. The data together with the fit are shown in Figures 3, 4, and 5. The parameters of the fits are listed in the captions of the corresponding figures.

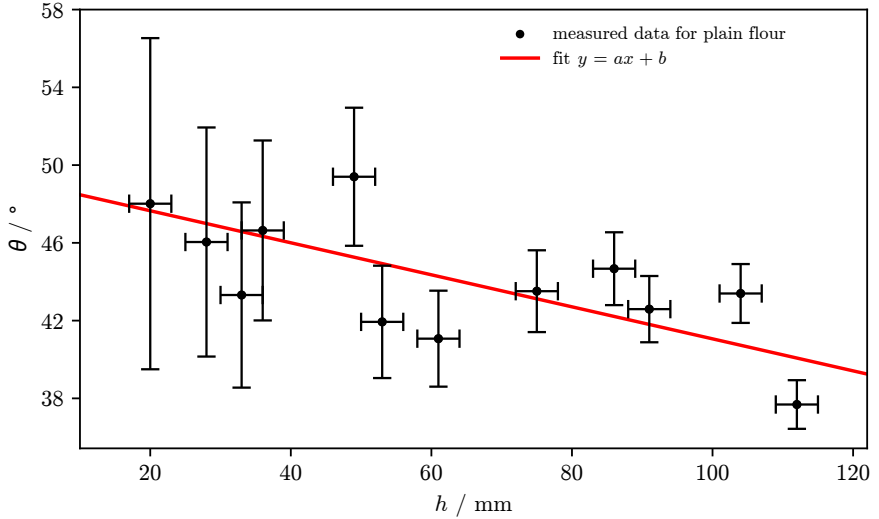


Figure 3: Plot of $\theta(h)$ —angle of repose versus cone height for flour. Fit parameters: $a = (-0.08 \pm 0.03)^\circ \cdot \text{mm}^{-1}$, $b = (49 \pm 3)^\circ$.

The average angles of repose for flour and poppy seeds were calculated as

$$\bar{\theta} = \frac{1}{n} \sum_{i=1}^n \theta_i,$$

with the sample standard deviation

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (\theta_i - \bar{\theta})^2},$$

and the uncertainty of the mean

$$\sigma_{\bar{\theta}} = \frac{s}{\sqrt{n}}.$$

The final uncertainty of θ was determined as $\sqrt{\sigma_{\bar{\theta}}^2 + \sigma_s^2}$, where σ_s is the maximum individual uncertainty from Eq. (4).

The resulting mean angles of repose were $\theta_{\text{flour}} = (44 \pm 9)^\circ$ for flour and $\theta_{\text{poppy}} = (31 \pm 4)^\circ$ for poppy seeds.

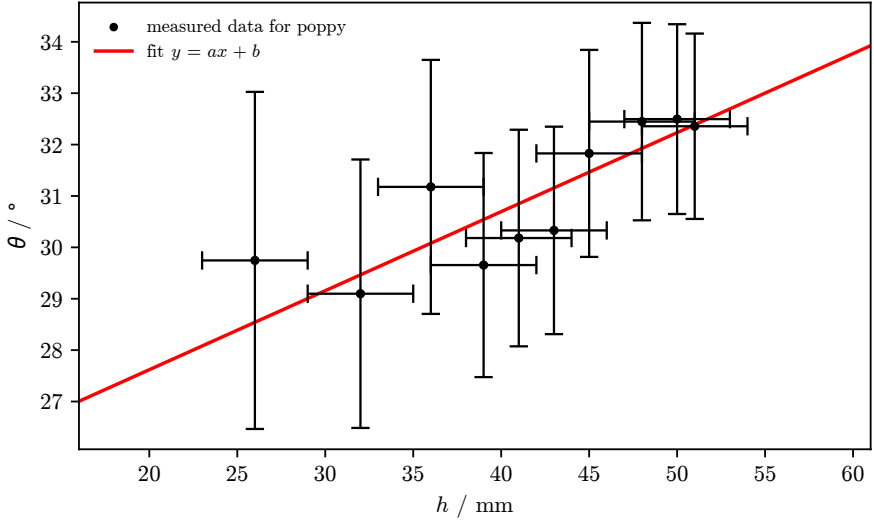


Figure 4: Plot of $\theta(h)$ —angle of repose versus cone height for poppy seeds. Fit parameters: $a = (0.15 \pm 0.03)^\circ \cdot \text{mm}^{-1}$, $b = (25 \pm 2)^\circ$.

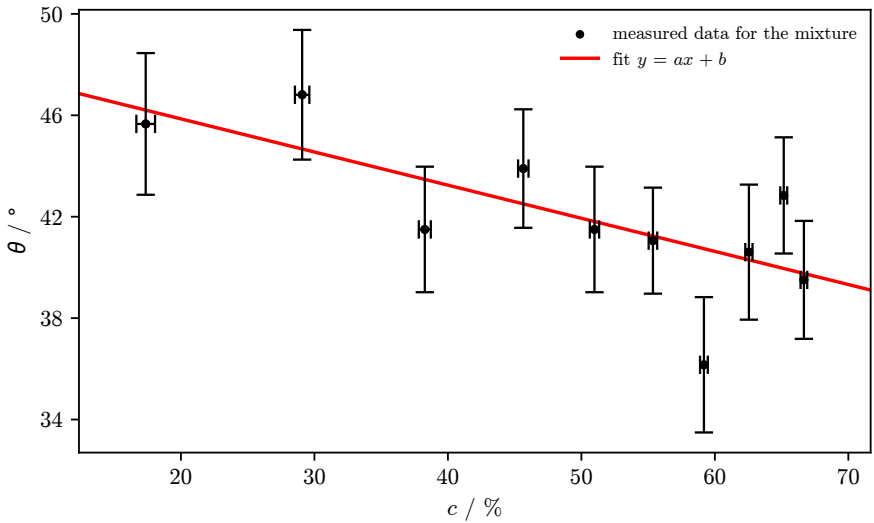


Figure 5: Plot of $\theta(c)$ —angle of repose versus mass concentration of poppy seeds. Fit parameters: $a = (-13 \pm 5)^\circ$, $b = (49 \pm 3)^\circ$.

Discussion

We first focus on the measurements for pure flour and pure poppy seeds. For fine flour, a literature value of the angle of repose is $(49 \pm 2)^\circ$ ¹. Our value overlaps only due to the relatively large measurement uncertainty (up to 20%). The mean differs by about 10%, as discussed below.

For poppy seeds, the theoretical value is slightly lower, around 26° ². Here, the measurement was more precise (uncertainty about 13%), though still differing by approximately 19% from the theoretical value.

The main sources of these deviations likely stem from the experimental apparatus, chosen for its availability in home conditions. The funnel opening was relatively wide, so the material did not fall exactly at one point as assumed in theory. Even with the funnel held close above the cone, the particles had enough velocity to roll off and even collapse the cone tip. Poppy seeds often bounced further away. Other possible causes include using different flour or poppy varieties than those in literature.

Secondary sources of error include reading distances inaccurately, imperfect cone shapes, and slight shaking of the funnel during measurement.

The main goal, however, was to study the dependence of the angle of repose on height. Both plots show that the fitted lines are not perfectly horizontal, but their slopes are small and have large relative uncertainties. More precise equipment and additional measurements would be needed for verification. Any deviation might be related to cohesive effects dependent on pressure or particle motion.

For mixtures, the angle of repose decreased with increasing poppy seed concentration, consistent with the initial assumption that poppy seeds have a lower angle. Verifying linearity would again require more data. Fit parameters suggest angles of about 49° for pure flour and 36° for pure poppy seeds—systematically higher than directly measured. Aside from previous causes, partial separation of the mixture was observed: most poppy seeds rolled to the lower cone region, as seen in Figure 6.



Figure 6: Cone formed during measurement for the mixture.

¹https://www.researchgate.net/publication/339839974_Analysis_of_static_angle_of_repose_with_respect_to_powder_material_properties

²<https://scispace.com/pdf/physical-properties-of-seeds-of-the-selected-oil-plants-1xluude3a5.pdf>

Finally, more precise methods can be mentioned. In practice, the so-called *static angle of repose* is often determined by slowly tilting a transparent container with the material until it begins to slide; the angle can then be measured directly. This minimizes distance-measurement errors. Another method involves lifting a container with a circular cross-section filled with material to form a cone—again simplifying distance measurement but requiring more equipment.

Conclusion

The measured mean angles of repose were

$$\theta_{\text{flour}} = (44 \pm 9)^\circ, \quad \theta_{\text{poppy}} = (31 \pm 4)^\circ,$$

and a dependence of the mixture's angle of repose on the mass concentration of both components was demonstrated. A weak dependence on cone height was also observed, which should be verified with more precise measurements.

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