Problem IV.2 ... boring Earth

3 points; (chybí statistiky)

The long forgotten gods of old became very bored while observing Earth. They therefore decided to transform the spherical Earth into a cylinder. The cylinder's axis of rotation passes through the center of its base and is perpendicular to it. What will be the ratio of the day length on the new Earth to the original day length? While they are gods, they are not magicians. Thus, the mass, density, and angular momentum of the cylinder remain the same as those of the original Earth. The height of the cylinder equals the diameter of the original Earth.

Lukáš was inspired by Saudi Arabia.

Let us start by establishing notation. Let R be the Earth's diameter, and r be the diameter of the cylinder. The height of the cylinder will be denoted by the letter h. Let L_1 and L_2 be the angular momentums of the round and cylindrical Earth, respectively. Similarly, we will denote the angular velocities by ω_1 and ω_2, and the period (the day length) will, again, be T_1 and T_2.

The spherical moment of inertia takes a form $J_1 = 2mR^2/5$, while the moment of inertia for a cylinder can be calculated as $J_2 = mr^2/2$.

Let us use the fact the angular momentum remains constant here, so we get

$$L_1 = L_2\,, \quad \Rightarrow \quad J_1\omega_1 = J_2\omega_2\,,$$
$$\frac{2}{5}mR^2\omega_1 = \frac{1}{2}mr^2\omega_2\,,$$
$$\frac{2}{5}R^2\frac{2\pi}{T_1} = \frac{1}{2}r^2\frac{2\pi}{T_2}\,,$$
$$\frac{T_2}{T_1} = \frac{5}{4}\frac{r^2}{R^2}\,.$$

The density remains constant as well. We, therefore, obtain

$$\rho_1 = \rho_2 \quad \Rightarrow \quad \frac{m}{V_1} = \frac{m}{V_2} \quad \Rightarrow \quad V_1 = V_2\,, \quad \Rightarrow$$
$$\Rightarrow \quad \frac{4}{3}\pi R^3 = \pi r^2 h \quad \Rightarrow \quad \frac{r^2}{R^2} = \frac{4}{3}\frac{R}{h}\,.$$

We substitute into the formula derived from the conservation of angular momentum

$$\frac{T_2}{T_1} = \frac{5}{4}\cdot\frac{4}{3}\frac{R}{h} = \frac{5}{3}\frac{R}{h}\,.$$

By substituting in the height h, we get the final ratio of the day lengths as

$$\frac{T_2}{T_1} = \frac{5}{6}\,.$$

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