### Problem III.E ... salted

12 points

Determine the specific heat capacity of kitchen salt.

Jarda is cooling his tea with salt.

### Introduction

The specific heat capacity c characterizes the ability of a body to change its temperature depending on its mass and the amount of heat supplied. The dimension of this quantity is  $[c] = J \cdot kg^{-1} \cdot K^{-1}$ ; the specific heat capacity therefore determines the amount of heat Q required to heat a body of mass 1 kg by 1 °C. It is a material property, and thus it differs from one material to another. We very often encounter the specific heat capacity of water  $c_{\rm H_2O} = 4180 \, {\rm J\cdot kg^{-1} \cdot K^{-1}}$ . This value is quite high in comparison with other materials—metals have specific heat capacities in the range of 100–500  ${\rm J\cdot kg^{-1} \cdot K^{-1}}$ . Therefore, a large amount of heat is needed to heat water, which can be exploited, for example, in central heating, as water retains a large amount of energy that it can then transfer to the radiator and the entire room.

# Theory

To determine the specific heat capacity of salt, we will start with the calorimetric equation

$$Q_{\rm in} = Q_{\rm out}$$
.

It describes the exchange of heat between two systems—the amount of heat released  $Q_{\text{out}}$  by the first system must equal the amount of heat received  $Q_{\text{in}}$  by the second system. To determine the individual heats, we use

$$Q = mc(t - t_0),$$

where m denotes the mass of the body,  $t_0$  the initial temperature, t the final equilibrium temperature, and c the specific heat capacity.

We will conduct the entire experiment in a calorimeter, which is a container isolated from the surroundings and generally characterized by its heat capacity  $C_{\rm k}$ . Salt is easily available in the form of loose crystals. However, if we conducted the experiment using only the crystals and the calorimeter, the air, being a thermal insulator, would not allow for uniform heating of the entire system, and we would not be able to use the above-mentioned oversimplified formula. We will, therefore, first mix the salt with oil since it practically does not dissolve in the oil, and we can thus neglect any additional chemical reactions that might occur if we, for example, mixed the salt with water.

The principle of our measurement lies in examining the heat transfer between three materials – the solid salt, the oil, and the calorimeter. The initial equation thus takes the form

$$\begin{split} Q_{\rm o} &= Q_{\rm k} + Q_{\rm s} \,, \\ m_{\rm o} c_{\rm o}(t-t_{\rm o}) &= C_{\rm k}(t_{\rm s}-t) + m_{\rm s} c_{\rm s}(t_{\rm s}-t) \,. \end{split}$$

Here, we assume that the oil will heat the salt and the calorimeter. By expressing the resulting temperature as

$$t(t_{\rm o}) = \frac{m_{\rm o}c_{\rm o}t_{\rm o} + m_{\rm s}c_{\rm s}t_{\rm s} + C_{\rm k}t_{\rm s}}{m_{\rm o}c_{\rm o} + m_{\rm s}c_{\rm s} + C_{\rm k}},$$

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we obtain the dependence of the resulting temperature on the oil temperature with two variables:  $c_s$  and  $C_k$ . We will try to keep the remaining quantities with the smallest possible variation. Finally, we assume that the specific heat capacity does not depend on temperature and that no substance undergoes a phase change.

## Setup and Execution of the Experiment

Before the experiment, we prepared an ice bath in a bowl to achieve the smallest possible differences in initial temperature  $t_s$ . We added water, ice, five bags of salt with a mass of  $m_s = 100 \,\mathrm{g}$ , and a thermos to a bowl. We then waited until the temperature of the mixture stabilized. Before starting the measurement, we measured a temperature in the bath ranging from 0 to 1 °C. We assume that the temperatures of the salt bags and the thermos also stabilized at the same values.

Our thermos is made of 18/8 stainless steel, i.e., with 18% chromium and 8% nickel content. Its specific heat capacity<sup>1</sup> is  $c_k = 0.5 \,\mathrm{kJ} \cdot \mathrm{^{C}}^{-1} \cdot \mathrm{kg}^{-1}$ . For the specific heat capacity of sunflower oi<sup>2</sup>, we use the value  $c_0 = (2.0 \pm 0.2) \,\mathrm{kJ} \cdot \mathrm{^{C}}^{-1} \cdot \mathrm{kg}^{-1}$ .

We performed the experiment for a total of five initial oil temperatures  $t_0$ , see Table 1. We carried out the first experiment at room temperature  $t_{01} = 18.3$  °C, which was the equilibrium temperature of a freely standing container with oil of mass  $m_0 = 100$  g. We placed the salt into the dried thermos, poured in the oil, and mixed everything using a thermometer. The resulting temperatures (along with their uncertainties) are listed in Table 1.

Table 1: Initial oil temperature  $t_0$  and resulting temperature of the salt and oil mixture t.

$\frac{t_o}{^{\circ}\mathrm{C}}$	$\frac{t}{\circ C}$
$18 \pm 1$	$12 \pm 1$
$55 \pm 3$	$34 \pm 2$
$130 \pm 7$	$74 \pm 4$
$186 \pm 9$	$102 \pm 5$
$202 \pm 10$	$135 \pm 7$

We proceeded similarly for the remaining four measurements. Prior to each, we heated the oil in an enamel pot on an induction stove. Once a certain temperature was exceeded, we removed the pot from the stove and waited for the thermometer reading to stabilize. We then poured the hot oil into the thermos containing the prepared salt. Between each measurement, we immersed the thermos in the ice bath so that its temperature matched that of the salt bags. Since the thermos was metallic, we assume the temperature stabilized relatively quickly.

The resulting temperatures can be seen in Table 1 and figure 1. We fitted the measured values with the theoretical function. Given that the masses of the oil and the salt are the same, that is,  $m=m_s=m_o=100\,\mathrm{g}$ , and that both the salt and the thermos were in the same ice bath, we assume  $t_s=0\,^{\circ}\mathrm{C}$ . After expanding the calorimeter's heat capacity as  $C_k=c_km_k$ , we obtain

$$t(t_{\rm o}) = \frac{c_{\rm o}t_{\rm o}}{c_{\rm s} + c_{\rm o} + c_{\rm k}\frac{m_{\rm k}}{m}}.$$

 $<sup>^{1} \</sup>mathtt{https://www.amardeepsteel.com/blog/18-8-stainless-steel.html}$ 

<sup>&</sup>lt;sup>2</sup>https://doi.org/DOI:10.1007/s10973-005-0050-x

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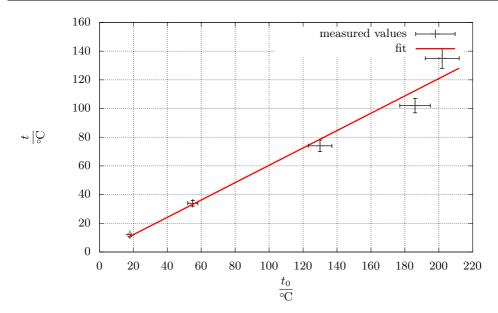


Figure 1: Dependence of the stabilized mixture temperature t on the initial oil temperature  $t_0$  with a linear fit.

The mass of the calorimeter  $m_k = (85 \pm 2)$  g was determined by measuring the submerged part of the thermos, estimating its thickness, and estimating the density of the steel used. By fitting the data with the function  $t(t_0)$  and substituting the known parameters, we obtain the line in Figure 1 and the final value of the specific heat capacity of salt as

$$c_{\rm s} = (0.9 \pm 0.1) \,\mathrm{kJ \cdot ^{\circ} C^{-1} \cdot kg^{-1}}$$
.

### Discussion

The main source of uncertainty during the measurement was undoubtedly the kitchen thermometer. The thermometer did not have a sufficiently fast readout rate, which was most apparent when heating the oil as after removing the enamel pot from the stove, the temperature of the oil measured by the thermometer continued to rise for a while before stabilizing and finally starting to decrease again. Since we do not know how delayed the measurement is, and because the manufacturer does not specify the uncertainty of the thermometer, we decided to conservatively overestimate the relative measurement uncertainty to 5%. Very hot oil also rapidly loses heat to its surroundings, so during the measurement and specifically while pouring the oil into the thermos, we acted as quickly as possible.

At temperatures higher than  $100\,^{\circ}\mathrm{C}$ , we heard a sound characteristic of water evaporation, which was likely caused by moisture evaporating from within the salt. At higher temperatures, this contributes to a lower final temperature, although we assume the amount of water stored in the salt is small.

In the calorimetric equation, we should ideally include the influence of the thermometer itself. At the beginning of mixing, the thermometer typically had an initial temperature equal to that of the oil. However, given the thermometer's dimensions, we do not expect significant heat loss due to it.

Overall, we assume that the resulting temperatures are systematically underestimated due to the above-mentioned factors and heat losses to the surroundings. The main losses are likely caused by heating the air as not all of the heat  $Q_{\rm in}$  is used to heat the oil and the calorimeter. We could quantify this effect by introducing an additional parameter q into the equation  $qQ_{\rm in} = Q_{\rm out}$ . However, there is no guarantee that the efficiency q would remain the same for different initial temperatures  $t_{\rm o}$ . Lower resulting temperatures manifest as an overestimation of the specific heat capacity of the salt.

In the theoretical introduction, we assumed the specific heat capacities of the involved materials are constant. As can be seen from tabulated values, for example in the case of salt, the specific heat capacity increases approximately linearly with temperature. These changes are on the order of a few percent for the salt and around  $10\,\%$  for oil. More precisely, we should say that we measured the average specific heat capacity of salt over the temperature range  $10\,^{\circ}\mathrm{C}$  to  $130\,^{\circ}\mathrm{C}$ .

The measured value  $c_s = (0.9 \pm 0.1) \, \text{kJ} \cdot ^{\circ} \text{C}^{-1} \cdot \text{kg}^{-1}$  corresponds, within uncertainty, to the tabulated value for salt<sup>4</sup>  $c_{\text{s-tab}} = 853 \, \text{J} \cdot ^{\circ} \text{C}^{-1} \cdot \text{kg}^{-1}$ . The large spread in the result is due to the effects described above.

### Conclusion

Using the calorimetric method, we determined the specific heat capacity of salt by mixing sunflower oil and table salt, obtaining the value  $c_s = (0.9 \pm 0.1) \, \mathrm{kJ} \cdot \mathrm{C}^{-1} \cdot \mathrm{kg}^{-1}$ . This value corresponds, within statistical uncertainty, to the tabulated value at room temperature  $c_{s\text{-tab}} = 853 \, \mathrm{J} \cdot \mathrm{C}^{-1} \cdot \mathrm{kg}^{-1}$ . The main influence on the measurement was the rapidly cooling oil and the use of a kitchen thermometer. At higher temperatures, the temperature dependence of the heat capacity also played a role.

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<sup>&</sup>lt;sup>3</sup>https://webbook.nist.gov/cgi/cbook.cgi?ID=C7647145&Mask=2#Thermo-Condensed

<sup>4</sup>https://en.wikipedia.org/wiki/Sodium\_chloride\_(data\_page)