Problem III.P ... bread

9 points

We can squeeze bread quite well, as there are a lot of cavities filled with gas. Determine the inner surface of all such cavities in a sourdough loaf.

Jarda is studying at a Department of Surface (and Plasma Science).

Given the diversity of approaches taken by the participants in estimating the internal surface area of bread, it is nearly impossible to summarize all correct ideas in a single selected solution. Therefore, in addition to a chosen solution, this text also provides a summary of the main approaches and commentary on the submitted solutions.

Approaches to Solving the Problem

To begin with, note that the approaches listed below do not by any means cover all submitted solutions and that most high-quality submissions combine several of them. The purpose of this section is thus to provide a point-by-point summary of ideas that deserve further commentary.

Estimating the Volume of Pores The main idea behind most of the submitted solutions was to determine the internal surface area from the internal volume. The following ideas, for instance, appeared in the submissions:

1. **Measuring the volume of bread before and after compression.** Assuming that compressing the bread (entirely or partially) successfully removes all the air from the pores, this method can be used to estimate the volume of the cavities. To measure the volume before and after compression, participants suggested approximating the bread as an ellipsoid or submerging it in a liquid—in such cases, however, one must also consider the penetration of the liquid into the bread and potential chemical or other effects.

2. **Analysis of 2D cross-sections.** Participants took photographs of slices (using phones or scanners) and counted the pores manually on a chosen grid or semi-automatically using software. The resulting surface fractions of pores were then converted into volumetric values through reasoning. Some of the photos included in the submitted solutions are shown in Figure 1.

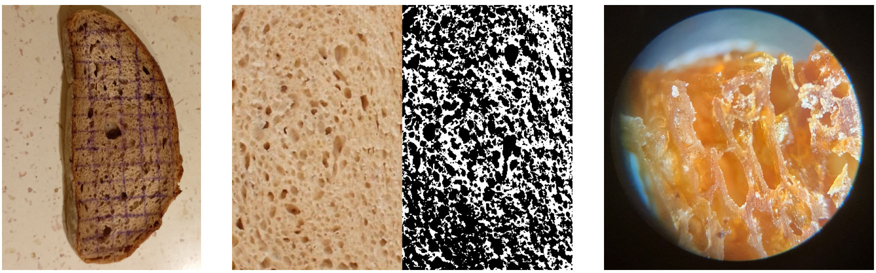

Figure 1: From the left: bread slice divided into sectors for manual cavity counting (Alena Marešová), photograph of a bread slice for computer analysis (Marco Komarnik), microscopic image of a bread slice (Tamara Dědková)

3. **Estimates from the literature.** Many participants also presented estimates based on studies or articles related to the topic. One particularly relevant example is the study *The bubble size distribution in wheat flour dough* [1], which directly addresses the relative frequency of different bubble sizes and also provides mathematical models.

A key source of uncertainty—aside from correctly determining the volume—is the choice of a *typical bubble* used for converting volume to surface area. Some participants, for simplicity, selected an average radius obtained from measuring pores or calculated based on values from the literature. However, this assumption neglects the fact that smaller bubbles contribute orders of magnitude less to volume than they do to surface area. For this reason, using a single radius is at least problematic, and participants who considered, for example, a range of bubble sizes in their calculations arrived at estimates that were more credible in this regard.

We also note that several participants correctly discussed aspects such as the shape of the cavities (*"Is it appropriate to assume they are spherical?"*), the distribution of the cavities (see paragraph above, *"How many and how big are the cavities contained in a bread?"*), and the definition of cavities themselves (since the bread's interior is essentially fractal in nature, it is appropriate to ask: *"What exactly qualifies as a cavity? What size threshold are we considering?"*), and so on.

Instrumental Methods As part of their research and analysis of the problem, some participants mentioned instrumental techniques that could be used to determine the surface area.

- **X-ray microtomography (μCT)** – a non-destructive, high-resolution method that was used, for example, in the previously mentioned article [1].
- **Magnetic Resonance Imaging (MRI)** – a gentle method without ionizing radiation; however, its resolution is lower, and it requires longer measurement times [2].
- **Mercury porosimetry** – a method for determining pore sizes based on the intrusion of mercury (or another non-wetting liquid) under high pressure that counteracts capillary pressure. For further information on this method, see the studies [3] and [4], which deal with the use of porosimetry (specifically in the context of measuring the porosity of baked goods!) and point out that this method is unable to measure cavities significantly larger than tenths of a millimeter in diameter.
- **Optical profilometers or laser 3D scanners** – other instrumental methods that can be used, for instance, for precise measurements of the internal structure of bread slices.

Theoretical Models A few participants attempted to approach the problem purely mathematically. They focused, for example, on estimating a specific mathematical distribution of bubbles in bread or on considerations of the fractal dimension of bread. However, this is not the place to outline solutions based on more complex mathematical models.

Example Solution

Below, we present a sample solution by Michal Stroff, which, despite apparent weaknesses (e.g., lacking a literature review or somewhat arbitrary parameter estimates[1]), effectively combines an intuitive and precise physical approach to the situation and yields a high-quality order-of-magnitude estimate.

To begin tackling the problem, we can make a few assumptions. One of them is that the internal cavities in bread have a spherical shape, which is a fairly valid assumption for an order-of-magnitude estimate. Suppose there are n such bubbles in a loaf of bread, and their root-mean-square radius is r. The total surface area of the internal cavities is then

$$S = 4\pi n r^2 \,.$$

The total volume of the bread will limit the total volume of cavities;

$$V_{\mathrm{bread}} = \frac{4}{3}\pi n r^3 \,.$$

In reality, the space between the spherical gaps must also be filled with dough, but this part is negligible given our rough approximations. Next, assume that all cavities are approximately the same size. If yeast is evenly distributed in the dough, there is no reason for larger bubbles to form in some areas during rising.

If we model the dough around the bubbles as a fluid with surface tension σ, smaller bubbles will collapse upon colliding with larger ones due to their higher internal pressure p. Recall that

$$p = \frac{2\sigma}{r} \,,$$

which clearly shows that small bubbles are overpressurized compared to larger ones and will release their contents into them. Furthermore, we show that collisions will primarily occur between bubbles with significantly different radii. If we model the motion of bubbles through the dough as laminar (low velocity, high viscosity, low Reynolds number), the bubble rises at a velocity

$$v \propto r^2 \,,$$

which is derived from the Stokes relation $F = 6\pi\eta r v$. That is, large bubbles rise much faster than small ones. This means that similarly sized bubbles will not collide as much, but large ones will effectively (due to their size) collect the smaller ones. Their speed is not high enough for the bubbles to escape from the surface of the dough into the surrounding environment. I estimate they move a few millimeters, which is sufficient to catch the smaller bubbles. Why is it important that small bubbles disappear? Simply because, as we will soon see, the smallest bubbles contribute the most to the surface area S per unit volume (a principle used in nature, e.g., in fat digestion). We cannot neglect them due to macroscopic indistinguishability, because they have a greater impact on the total surface area than the visible bubbles. Fortunately, they should disappear, so we do not need to worry about them. This also lends greater validity to the assumption

$$\sqrt[3]{\langle r^3 \rangle} = \sqrt{\langle r^2 \rangle} \equiv r \,.$$

[1] This is not a significant problem given the overall goal of obtaining an *order-of-magnitude* estimate.

We thus consider the two radii appearing in the formulas for V_{bread} and S to be the same. These two formulas can then be combined into one

$$S \sim \frac{3V_{\text{bread}}}{r}\,.$$

We see that smaller bubbles increase the internal surface area (as mentioned earlier). We can now estimate the internal surface area of the bread cavities as

$$S \sim \frac{L_{\text{bread}}^3}{r}\,,$$

where L_{bread} denotes the characteristic dimension of a Šumava-style bread loaf, e.g., its width $L_{\text{bread}} = 20\,\mathrm{cm}$. The bubble radius $r = 2\,\mathrm{mm}$ can be estimated by visually inspecting the internal bread structure. Then we get:

$$S \sim \frac{0.2^3}{0.002}\,\mathrm{m}^2 = 4\,\mathrm{m}^2\,.$$

So, the surface area of the bread's internal cavities is, in order of magnitude, a few square meters.

Results

Finally, for curious readers, we present a histogram in Figure 2, which shows a summary of the results obtained by the participants. It is remarkable that most of them—despite the diversity of the approaches used (and the varying degree of errors in these approaches ⌣̈)—agreed at least in order of magnitude. Based on these data, the readers can thus form their idea of the internal surface area of bread.

References

[1] P. S. Kadan, E. A. Robinson, and J. B. Thibodeau, "Characterization of texture of bread using compression testing and stress–relaxation," *Food Research International*, vol. 39, no. 3, pp. 265–272, 2006. https://doi.org/10.1016/j.foodres.2006.07.020

[2] S. F. Bader, S. E. Taylor, and J. H. Hotchkiss, "Proton nuclear magnetic resonance imaging of bread dough during proofing and baking," *Cereal Chemistry*, vol. 80, no. 4, pp. 390–395, 2003. https://doi.org/10.1094/CCHEM.2003.80.4.390

[3] M. E. Flores-López and R. G. Labavitch, "Application of mercury porosimetry to the study of bread crumb structure," *Food Microstructure*, vol. 11, no. 2, article 4, 1992. https://digitalcommons.usu.edu/foodmicrostructure/vol11/iss2/4

[4] A. N. Tihonov and V. V. Varlamov, "Application of porosimetry and fractal analysis in food structure investigation," *Applied Mathematical Modelling*, vol. 35, no. 9, pp. 4352–4361, 2011. https://doi.org/10.1016/j.apm.2011.03.047

Vojtěch David
vojtech.david@fykos.org

FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of CUNI MFF, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports of the Czech Republic.

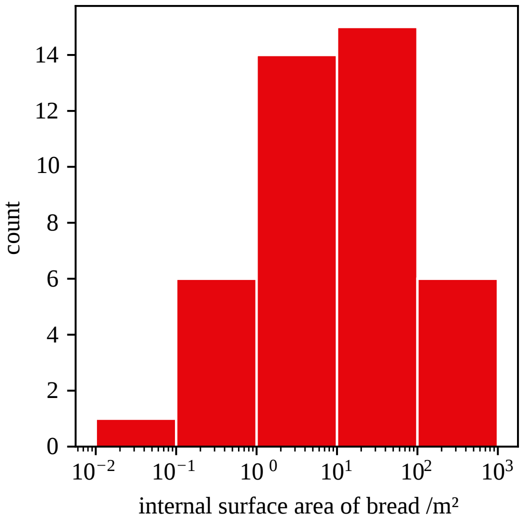

Figure 2: Summary of results.