## Problem III.1 ... lazy river

3 points; průměr 2,41; řešilo 115 studentů

*Jarda is lying on his pool float in a water park and he is being carried away by an artificial river current, when he notices his friend swimming toward him, intending to capsize Jarda. His friend is swimming at speed* $0.5\,\mathrm{m\cdot s^{-1}}$ *relative to the current. When the distance between Jarda and his friend is only* $3\,\mathrm{m}$*, the cross-section area of the artificial river bed narrows from* $4\,\mathrm{m^2}$ *to* $3\,\mathrm{m^2}$*. How much time does Jarda have to prepare for the capsize if the initial current speed was* $0.8\,\mathrm{m\cdot s^{-1}}$*?* *Target destroyed. Enemy vessel neutralized.*

Let us denote the initial velocity of the river by $v_1$. The velocity of Jarda's friend is given by the sum of his own velocity $v'_\mathrm{k}$ and the current velocity, i.e., $v_\mathrm{k} = v'_\mathrm{k} + v_1$. Let us start measuring the time when Jarda and his buddy pass into the narrower part of the river. The time it takes Jarda's friend to reach the interface with the narrower part of the river is given as

$$t_1 = \frac{s_1}{v_\mathrm{k}} = \frac{s_1}{v'_\mathrm{k} + v_1} \doteq 2.31\,\mathrm{s}\,,$$

where $s_1 = 3\,\mathrm{m}$ is the initial distance between Jarda and his friend. The continuity equation yields that the flow rate in both parts of the river must be the same at the same time. This can be expressed mathematically as

$$S_1 v_1 = S_2 v_2\,.$$

From this, we can express the velocity $v_2$ in the narrower part of the river as

$$v_2 = \frac{S_1}{S_2} v_1\,.$$

However, in the time $t_1$, the river carries Jarda away by distance

$$s_2 = t_1 v_2\,.$$

Jarda's buddy will have to overcome this distance now. Consider a coordinate system associated with the river, i.e., one in which Jarda is at rest and his friend moves with velocity $v'_\mathrm{k}$. The time it takes Jarda's friend to cover the distance is

$$t_2 = \frac{s_2}{v'_\mathrm{k}} = \frac{S_1}{S_2} \frac{s_1 v_1}{(v'_\mathrm{k} + v_1)\, v'_\mathrm{k}} \doteq 4.92\,\mathrm{s}\,.$$

The final result is, therefore, the sum of the two previous times, so

$$t = t_1 + t_2 \doteq 7.23\,\mathrm{s}\,.$$

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FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of CUNI MFF, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports of the Czech Republic.