## Problem II.E ... tension in the kitchen

12 points; průměr 7,21; řešilo 78 studentů

*Measure the deformation curve for an ordinary rubber band.* *Jarda worries he'll go ballistic.*

### *Theoretical Introduction*

The deformation of a body is a change in its dimensions, shape, or volume caused by external forces. Deformations can be classified into two types:

- elastic (reversible), which disappear once the external forces are removed;
- plastic (permanent), where the body remains deformed even after the external forces cease to act.

There are several ways in which a body can be deformed, such as tension, compression, bending, shear, and torsion. In this task, we will consider only deformation caused by tension.

A fundamental quantity describing deformation is normal stress $\sigma$, which is analogous to pressure in fluids. It is defined as the force $F$ acting perpendicularly on the surface $S$ of the body, expressed as

$$\sigma = \frac{F}{S}\,.$$

To describe deformation, we also introduce the relative elongation $\varepsilon$:

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0}\,,$$

where $\Delta l$ is the absolute elongation of the rubber band, $l_0$ is its original length, and $l$ is its current length. For elastic deformation under tension, Hooke's law states that the stress in a deformed body is proportional to the relative elongation

$$\sigma = E\varepsilon\,,$$

where $E$ is Young's modulus, a material constant.

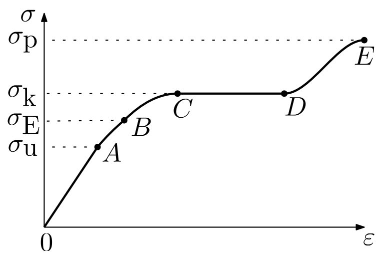


Figure 1: General deformation curve.

In figure 1, the general deformation curve is divided into several sections: the segment from 0 to $A$ corresponds to elastic deformation (Hooke's law holds). After exceeding the proportional limit $\sigma_\mathrm{u}$, the section between $A$ and $B$ represents anelastic deformation. Anelastic deformation is still reversible, but after unloading, the material returns to its original state only after some time. Beyond the elastic limit $\sigma_E$, plastic deformation occurs. In the section between $C$ and $D$, the material undergoes what is known as creep, where exceeding the yield strength $\sigma_\mathrm{k}$ results in a large relative elongation for a small stress change. Between $D$ and $E$, the material undergoes strain hardening, and beyond the ultimate tensile strength $\sigma_\mathrm{p}$, the material fractures.

However, we will focus on the general relationship between $\sigma$ and $\varepsilon$, even for cases of inelastic deformation. When this dependence is plotted on a graph, the resulting curve is called the deformation curve. The general deformation curve is shown in figure 1. Different materials exhibit different deformation curves, and the proportions of the individual sections may vary, or some sections may be entirely absent.

### *Experiment Description*

To measure the deformation curve of a kitchen rubber band, we used the following equipment:

- kitchen rubber band,
- digital microscope Hüntermann HMI-05U (with an accuracy of 0.002 mm),
- measuring tape (with a 1 mm graduation),
- kitchen scale First Austria (with an accuracy of 1 g),
- water bucket with a thin handle,
- waterproof bag,
- syringe.

For greater accuracy, we do not cut the rubber band. Instead, we determine its initial length $l_0$ as half of its total length. Both the bucket handle and the attachment point of the rubber band are considered thin enough to neglect the band's length in the bending region and assume it remains the same for all measurements. The rubber band is then hung vertically, and a bucket of mass $m_\mathrm{k}$ is attached to its lower end. At this moment, the rubber band is stretched to length $l$ by the weight $W = m_\mathrm{k} g$ acting downward. The relative elongation of the rubber band is defined as

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0}\,.$$

If we assume a cross-sectional area $A$ along the entire rubber band, we can express the normal stress in the rubber band caused by the weight $W$ as

$$\sigma = \frac{W}{2A} = \frac{m_\mathrm{k} g}{2A}\,,$$

where we multiplied the cross-sectional area by a factor of two because in the case of an uncut rubber band, the bucket is suspended by two strands.

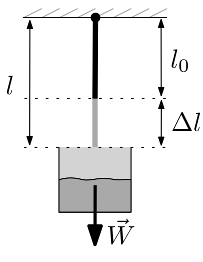


Figure 2: Simplified diagram of the apparatus.

When we add water of volume $V$ and density $\rho$ to the bucket, the rubber band stretches to a length $l'$, and we now have

$$\varepsilon' = \frac{\Delta l'}{l_0} = \frac{l' - l_0}{l_0}\,, \qquad \sigma' = \frac{(m_\mathrm{k} + \rho V)g}{2A}\,.$$

We continue this process to measure the deformation curve of the rubber band until it breaks.

To verify Hooke's law, we primarily need to measure the relationship for small relative elongations. However, the bucket itself may be too heavy and stretch the rubber band more than desired. Therefore, for the first few measurements, we use a lighter waterproof bag instead of the bucket. The measurement process remains the same, and we use the same formulas for $\varepsilon'$ and $\sigma'$, except that we replace the bucket mass $m_\mathrm{k}$ with the waterproof bag mass $m_\mathrm{p}$.

### *Data Processing*

In our calculations, we assume the gravitational acceleration $g = 9.81\,\mathrm{m{\cdot}s^{-2}}$ and the density of water $\rho = 1.000\,\mathrm{kg{\cdot}m^{-3}}$.

Using a measuring tape, we determined the initial length of the rubber band (half of its total length) as $l_0 = (50.2 \pm 0.5)\,\mathrm{mm}$, where the uncertainty was estimated as half the size of the smallest part of the scale. This uncertainty is considered for all length measurements taken with the tape.

Using a scale, we determined the mass of the bucket to be $m_\mathrm{k} = (110 \pm 1)\,\mathrm{g}$ and the mass of the waterproof bag to be $m_\mathrm{p} = (17 \pm 1)\,\mathrm{g}$.

Using a microscope, we measured the dimensions of the rubber band and estimated its cross-sectional area as a rectangle with sides $a = 1.40\,\mathrm{mm}$ and $b = 1.47\,\mathrm{mm}$. The measurement uncertainty $\Delta a = \Delta b = 0.03\,\mathrm{mm}$ was significantly overestimated based on microscope calibration we did by using a calibration plate of known length and considering slight variations in these dimensions along the rubber band. The final cross-sectional area of the rubber band was determined as $A = ab$, with uncertainty calculated according to the uncertainty propagation formula for the product of two quantities:

$$\frac{\Delta A}{A} = \sqrt{\left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta b}{b}\right)^2}\,,$$

resulting in $A = (2.06 \pm 0.06)\,\mathrm{mm}^2$. Similarly, we calculated the uncertainties $\Delta\varepsilon$ and $\Delta\sigma$.

Water was added using a syringe with a graduation of 5 ml, corresponding to a mass of 5 g. For time efficiency, several "divisions" of water were added at once. The uncertainty in volume was neglected.

A total of 94 measurements were taken, with the first three using the waterproof bag. For clarity, only the first 10 and the last 3 values are shown in Table 1.

### *Results and Discussion*

In the graph in figure 5, it is difficult to precisely determine the proportionality limit $\sigma_\mathrm{u}$, the elastic limit $\sigma_E$, and the yield point $\sigma_\mathrm{k}$. However, we can at least estimate them as:

$$\sigma_\mathrm{u} \approx 0.3\,\mathrm{MPa}\,, \qquad \sigma_\mathrm{E} \approx 0.5\,\mathrm{MPa}\,, \qquad \sigma_\mathrm{k} \approx 0.8\,\mathrm{MPa}\,.$$


Figure 3: Dimension $a$ of the rubber band.

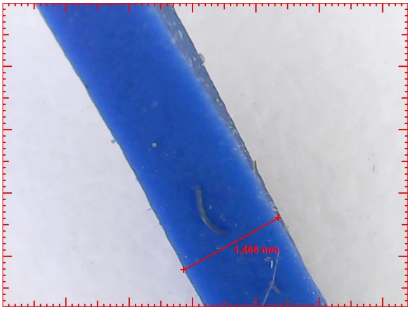


Figure 4: Dimension $b$ of the rubber band.

Table 1: Table of measured values.

| $N$ | $\frac{m}{\text{kg}}$ | $\frac{l}{\text{mm}}$ | $\frac{\Delta l}{\text{mm}}$ | $\frac{\varepsilon}{1}$ | $\frac{\sigma}{\text{MPa}}$ | $\frac{\Delta\varepsilon}{1}$ | $\frac{\Delta\sigma}{\text{MPa}}$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.017 | 50.7 | 0.5 | 0.01 | 0.04 | 0.0003 | 0.003 |
| 2 | 0.047 | 52.2 | 2.0 | 0.04 | 0.11 | 0.0012 | 0.004 |
| 3 | 0.082 | 54.2 | 4.0 | 0.08 | 0.19 | 0.003 | 0.007 |
| 4 | 0.11 | 56.2 | 6.0 | 0.12 | 0.26 | 0.003 | 0.008 |
| 5 | 0.13 | 58.2 | 8.0 | 0.16 | 0.31 | 0.004 | 0.009 |
| 6 | 0.15 | 60.2 | 10.0 | 0.20 | 0.36 | 0.005 | 0.011 |
| 7 | 0.17 | 62.2 | 12.0 | 0.24 | 0.41 | 0.006 | 0.012 |
| 8 | 0.19 | 64.2 | 14.0 | 0.28 | 0.45 | 0.007 | 0.014 |
| 9 | 0.21 | 67.2 | 17.0 | 0.34 | 0.50 | 0.008 | 0.015 |
| 10 | 0.23 | 69.7 | 19.5 | 0.39 | 0.55 | 0.009 | 0.016 |
| ... | ... | ... | ... | ... | ... | ... | ... |
| 92 | 1.87 | 297.2 | 247 | 4.92 | 4.46 | 0.05 | 0.13 |
| 93 | 1.89 | 298.2 | 248 | 4.94 | 4.50 | 0.05 | 0.13 |
| 94 | 1.91 | 299.2 | 249 | 4.96 | 4.55 | 0.05 | 0.13 |

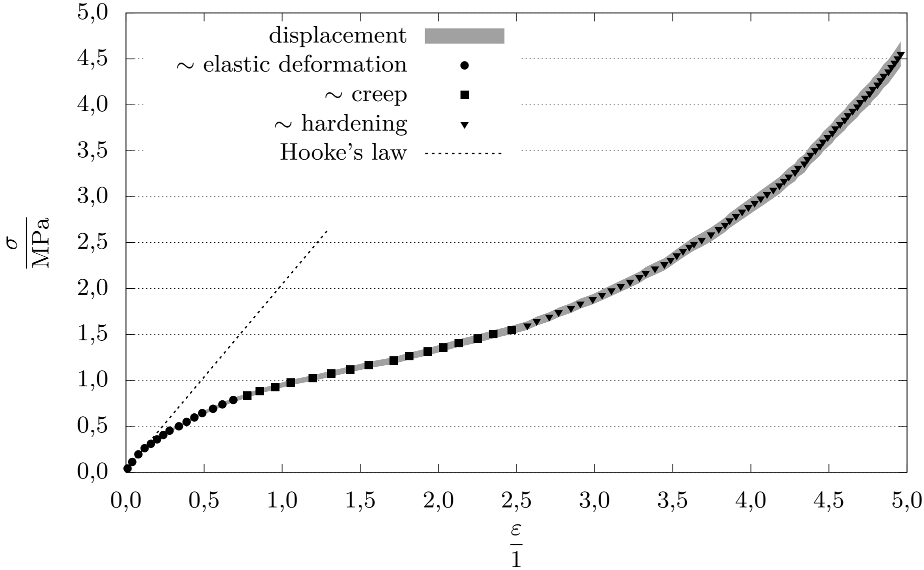


Figure 5: Measured deformation curve.

To avoid influencing the mechanical properties of the rubber band too much, we did not remove the load between measurements and did not test the effect of elastic recovery. However, we see that the first few values lie approximately on a straight line, which corresponds to Hooke's law. Therefore, we fitted the first four values in the program `gnuplot` with the equation $\sigma(\varepsilon) = E\varepsilon + \sigma_0$ and obtained the following parameters:

$$E = (2.02 \pm 0.10)\,\mathrm{MPa}\,, \qquad \sigma_0 = (0.027 \pm 0.008)\,\mathrm{MPa}\,.$$

The value of $E$ is Young's modulus for tensile deformation of our rubber band, and it corresponds well with the range of 1.5, MPa to 5, MPa for rubber (as per standard tables). The constant $\sigma_0$ represents a combination of the stress induced by the weight of the rubber band, which weighs approximately 1 g, as well as the error due to approximations and simplifications we made during the analysis.

We can further observe that in the region of inelastic deformation, the points marked with squares are less densely packed, even though the weight increment between data points remains constant. This suggests that material flow may be occurring, where the same change in stress results in a greater elongation. Finally, we see that at the end of the curve, the points become more densely packed, which corresponds to the material hardening. The curve ends with the rupture of the rubber band, indicating that the tensile strength for our particular rubber band is approximately:

$$\sigma_\mathrm{p} \approx 4.5\,\mathrm{MPa}\,.$$

The rubber band may have deformed differently than just by tension, for example, it might have slightly twisted or sheared. As seen in the images, the rubber band was not uniformly wide and appeared non-homogeneous. Of course, the cross-sectional area of the rubber band changed during stretching, even though we did not consider this when calculating the normal stress. Since the rubber band was not homogeneous, this effect was difficult to estimate. For a homogeneous material, the ratio of relative elongation to relative transverse contraction under tensile stress is given by Poisson's ratio, which for rubber has a numerical value of approximately 2.

### *Conclusion*

We measured the deformation curve for a typical kitchen rubber band, and this curve is plotted in the graph in figure 5.

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