

Problem II.P ... cold lovers' association

10 points

Let the quality of acclimatization to cold temperatures be defined based on the thermal power that a person must exert. Is it better to acclimatize under flowing water or by immersing oneself in a bathtub of cold water? Consider at least the different water temperatures, flow rate, and the temperature of the surroundings.

Michal took a cold shower.

Cold exposure training is a process in which the body loses heat to the surrounding environment, which stimulates the organism's thermoregulatory mechanisms. There are various methods of cold exposure training; in this problem, we discuss immersion in a bathtub of cold water and showering under running cold water. Each of these methods represents a different mechanism of heat transfer and therefore has different effects on the body's heat loss.

During immersion in a bathtub, the body is surrounded by water, which gradually warms up as a result of heat transfer from the body surface. This process is governed primarily by heat conduction through the skin. By contrast, during showering, running water constantly washes away the warmed layer of water from the body, maintaining a larger temperature difference, which implies more intensive heat removal. Here, heat transfer by convection plays the most significant role.

Our goal is to describe the physical laws that affect heat transfer in both methods and to describe the differences in terms of the thermal power that the body must produce to maintain its temperature. The factors we examine include water temperature, volumetric flow rate, and ambient temperature.

Heat transfer by conduction (Fourier's law)

Heat conduction is the dominant mechanism during immersion in water. The heat flux q transferred by conduction can be expressed by Fourier's law

$$q = -k \frac{dT}{dx},$$

where k is the thermal conductivity and $-dT/dx$ is the temperature gradient.

For a thin layer of skin of thickness d , the temperature gradient can be approximated as

$$-\frac{dT}{dx} = \frac{T_t - T_v}{d},$$

where T_t is the human body temperature and T_v is the water temperature. For the thermal power dQ/dt , we therefore express Fourier's law in the form

$$\frac{dQ}{dt} = qS = kS \frac{T_t - T_v}{d}, \quad (1)$$

where S is the contact area of the bodies between which heat transfer occurs (in our case, between the body and the water).

Thermoregulatory processes try to maintain a constant human body temperature T_t ; however, stagnant water in a bathtub, which is not replaced by flow, changes its temperature over time, so $T_v = T_v(t)$. The thermal power during water immersion, therefore, also changes over time. In a large volume of water, we can consider evaporation negligible and the volume constant. The thermal conductivity of air is small compared with that of human skin, so we neglect

heat transfer between the water and the ambient air. We consider only heat transfer between the body and the water, which is described by the calorimetric equation

$$dQ = c_v V_v \rho_v dT_v, \quad (2)$$

where c_v is the specific heat capacity of water, V_v is the water volume, and ρ_v is the density of water.

We differentiate Eq. (2) with respect to time and compare it with Eq. (1).

$$\begin{aligned} \frac{dQ}{dt} &= c_v V_v \rho_v \frac{dT_v}{dt}, \\ kS \frac{T_t - T_v}{d} &= c_v V_v \rho_v \frac{dT_v}{dt}. \end{aligned}$$

We denote

$$\begin{aligned} \alpha &= \frac{k S}{c_v V_v \rho_v d}, \\ \frac{dT_v}{dt} &= \alpha(T_t - T_v), \\ \frac{dT_v}{T_t - T_v} &= \alpha dt. \end{aligned}$$

We integrate the equation:

$$\begin{aligned} \int \frac{dT_v}{T_t - T_v} &= \int \alpha dt, \\ -\ln |T_t - T_v| &= \alpha t + C, \\ |T_t - T_v| &= \exp(-\alpha t - C). \end{aligned}$$

Since the body temperature T_t is higher than the temperature of the water in which the person is acclimatizing to cold, we can remove the absolute value.

$$T_t - T_v = K \exp(-\alpha t).$$

From the initial condition, when the water had temperature $T_v(0) = T_{v0}$ at time $t = 0$, we determine the constant K :

$$T_t - T_{v0} = K.$$

For the time evolution of the water temperature, we obtain

$$T_v(t) = T_t + (T_{v0} - T_t) \exp\left(-\frac{kS}{c_v V_v \rho_v d} t\right). \quad (3)$$

During heat exchange by conduction, the water therefore warms up; this depends on the initial water temperature $T_{v,0}$, its volume V_v , and the body area in contact with the water S . After substituting relation (3) back into Eq. (1), we obtain

$$\frac{dQ}{dt} = k S \frac{T_t - T_v(t)}{d} = \frac{k S}{d} (T_t - T_{v0}) \exp\left(-\frac{kS}{c_v V_v \rho_v d} t\right), \quad (4)$$

so the thermal power decreases over time.

A typical human body temperature is approximately $T_t = 310\text{ K}$ [1]. The average body surface area of an adult is $S = 1.86\text{ m}^2$ [2], and the thickness of human skin is $d = 4\text{ mm}$ [3]. Let us assume that during cold exposure training in a bathtub, the whole person is immersed; we neglect the area of the face that remains above the surface for breathing. The thermal conductivity of human skin and water is approximately the same and is $k = 0.6\text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$ [4]. The volume of an ordinary bathtub may be $180 - 220\text{ l}$ [5], and the volume of a human body is 65 l [6]; we therefore consider the water volume to be $V_v = 150\text{ l}$. The initial water temperature suitable for cold exposure training may be $T_v(0) = 280\text{ K}$. The specific heat capacity of water is approximately $c_v = 4200\text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$, and the density of water is $\rho_v = 1000\text{ kg}\cdot\text{m}^{-3}$ [7]. After substituting these values into relation (4), we obtain a thermal power of just under 8.4 kW at time $t = 0\text{ s}$. After five minutes of cold exposure, the thermal power decreases to approximately 7.4 kW , and after ten minutes to 6.4 kW .

Among the examined parameters, the initial water temperature has a large effect during cold exposure training in a bathtub. From relation (4), it is clear that the thermal power is directly proportional to the temperature difference ($T_t - T_{v0}$). This difference is one of the main driving mechanisms of heat transfer. The lower the water temperature, the faster the body loses heat, which increases the demands on the organism's thermoregulatory abilities. Low water temperatures cause rapid cooling of the body surface, which leads to the activation of several defense mechanisms. Blood vessels in the skin constrict, thereby limiting heat loss from the blood. At the same time, metabolic activity increases in order to generate more heat. The body thus responds to intense cold with increased thermal power, which is important for developing resistance to low temperatures. At higher water temperatures, the cooling process is slower and less intense, giving the body an opportunity for gradual adaptation. The lower the water temperature, the more intense the cold exposure training. However, this also requires greater adaptation from the body and may be more demanding. Higher water temperatures, by contrast, allow for more gradual and tolerable cold exposure training, but the effect may not be as pronounced. The ideal temperature depends on the goal of the cold exposure training and the individual's fitness.

Since we do not replace the water in the bathtub, the water flow rate is approximately zero and has no significant effect on this method of cold exposure training. During immersion in a bathtub, almost the entire body is surrounded by water, so the effects of the ambient temperature are also not significant. Cold air can remove heat from the skin on the exposed parts of the body, such as the face, and further contribute to the overall thermal stress.

Heat transfer by convection (Newton's law of cooling)

The water flow rate significantly affects the intensity of cold exposure training through how effectively it removes heat from the body surface. Running water can more quickly replace the warmed layer of water in immediate contact with the skin with new, cold water. This process increases the rate of heat transfer, which leads to greater heat loss and more intense cooling. At a low water flow rate, a partially warmed layer remains close to the body; this acts as a thermal insulator and slows cooling. By contrast, at a high flow rate, this layer is continuously removed, which increases the cooling effect. This mechanism makes cold exposure training under running water a more demanding method because the body must produce more heat to maintain a constant temperature. The water flow rate also allows greater flexibility during cold exposure training. People can regulate the flow rate: a lower flow rate is suitable

for gradual cold exposure training, while a higher flow rate provides more intense stimulation of the thermoregulatory system.

During showering, heat is therefore transferred primarily by convection; the thermal power in this case is described by Newton's law of cooling:

$$\frac{dQ}{dt} = hS (T_t - T_v), \quad (5)$$

where h is the heat transfer coefficient.

During showering, the running water is continuously replaced. Water warmed by the body is thus almost immediately replaced by new, cold water. Because of this continuous replacement, we can consider the water temperature T_v to be constant. During heat transfer by convection, the thermal power therefore remains constant as well.

The value of the heat transfer coefficient is approximately $h_v = 300 \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ for cold water and $h_o = 25 \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ for air [8]. However, the magnitude of the heat transfer coefficient depends on the flow speed and can reach significantly different values. The area of the body that is in contact with water during showering is not easy to determine; it is related, for example, to the volumetric flow rate of the shower head. At a higher volumetric flow rate, the water covers a larger area of the body. Suppose that the water covers 50% of the body surface; then the thermal power according to relation (5) is approximately 8.4 kW.

If we also take into account air flow at room temperature $T_o = 295 \text{ K}$, we obtain an additional contribution of 0.35 kW to the thermal power; the total thermal power during heat transfer by convection while showering is thus approximately 8.75 kW. If we keep the same stream of falling water during showering, this thermal power will remain constant throughout the entire cold exposure period.

In relation (5), we see that, as with cold exposure training in a bathtub, the thermal power during showering is directly proportional to the temperature difference ($T_t - T_v$), which has the same effect as in the previous method. In a shower, a smaller body area is in contact with the water, especially if an ordinary stream of water is used, which may hit only certain parts of the body. The water moves quickly and removes the warmed layer of water from the skin, but because contact is limited to only certain parts of the body (face, shoulders, arms, etc.), the heat losses are smaller than during immersion of the whole body. Nevertheless, running water can be effective for stimulating thermoregulatory responses, especially if the water flow rate is high, which increases the intensity of cooling in the given area and the body area on which the water falls.

If the bathroom or surrounding space is cold, the air can contribute significantly to the total heat loss. Cold air accelerates body cooling because it increases convection – the warm air in contact with the skin is replaced by cold air, which accelerates heat loss. During showering, therefore, a colder surrounding environment leads to more intense heat loss, while warmer surrounding air reduces the overall load on the body.

During immersion in a bathtub, the effect of heat transfer by convection also appears. For still water, the heat transfer coefficient is approximately $h_{v0} = 50 \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ [8]. In this case, we again assume that the water is in contact with the entire surface area of the human body. According to relation (5), the thermal power due to heat transfer by convection during immersion in a bathtub is approximately 2.8 kW. By adding the thermal power from heat transfer by conduction, we obtain a thermal power of 9.2 kW during immersion in a bathtub.

Heat transfer by radiation (Stefan–Boltzmann law)

On the remaining body surface that is not in contact with water, heat is transferred by radiation between the body and its surroundings. The thermal power of radiation is described by the Stefan–Boltzmann law

$$\frac{dQ}{dt} = \sigma S \varepsilon (T_t^4 - T_o^4), \quad (6)$$

where σ is the Stefan–Boltzmann constant, ε is the emissivity, and T_o is the ambient temperature.

Since water has a high heat capacity and density, heat transfer by radiation in a water environment is negligible compared with conduction and convection.

The value of the Stefan–Boltzmann constant is $5.67 \cdot 10^{-8} \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$, and the emissivity of the human body is $\varepsilon = 0.96$ [1]. Even if the entire surface area of the human body is in contact with the ambient air, the thermal power during heat transfer by radiation according to relation (6) is only 168 W; it is therefore negligible compared with heat transfer by conduction and convection.

Conclusion

Cold exposure training is an effective method for stimulating the thermoregulatory system of the human body. Different methods of cold exposure training – immersion in cold water and showering with cold running water – differ in their heat transfer mechanism, the intensity of the thermal load, and the dynamics of cooling. During immersion in a bathtub, heat transfer by conduction predominates; its intensity gradually decreases over time as the water warms up. By contrast, showering is characterized by convection, which ensures constant thermal power due to the continuous renewal of cold water at the body surface. A negligible amount of heat is removed by radiation. The key factor remains the temperature difference between the body and the water, which determines the difficulty of the entire process. The choice of method depends on the desired cooling intensity and the goals of cold exposure training.

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