Problem II.2 ... (not really) capillary phenomena

3 points; průměr 2,44; řešilo 144 studentů

Monika has attempted to create her own kind of tree. However, as a physicist, she disregarded all biological phenomena and aspects. For her tree, she took a long capillary tube with a diameter of $d = 0.1\,\mathrm{mm}$. How high does the water rise in this physics-based tree? For which fluid are such trees the tallest? Additionally, by neglecting which physical principle does the height of this "tree" differ so significantly from that of real trees?

Monika was trying to prove biology is not important.

We will calculate the level of the water column in the capillary tree by using formula for the height of the liquid column in a capillary tube of diameter d

$$h_{\mathrm{water}} = \frac{4\sigma}{gd\rho} \doteq 0.294\,\mathrm{m}\,,$$

where $\sigma \doteq 7.27 \cdot 10^{-2}\,\mathrm{N{\cdot}m^{-1}}$ is the surface tension of water, $\rho \doteq 1\,000\,\mathrm{kg{\cdot}m^{-3}}$ its approximate density and $g \doteq 9.81\,\mathrm{m{\cdot}s^{-2}}$ gravitational acceleration. Water in such a tree could rise to approximately 29.4 cm.

The calculated height of the tree is remarkably different from the height of real trees due to the evident neglect of all biological factors. It also ignores evaporation from the peripheral parts of the tree (primarily the leaves), which is the main driving force behind the upward movement of the water column in the capillary. This phenomenon affects real tree height more than the capillary rise.

To determine which liquid would give our tree the highest possible height, we have to discuss the quantities in the above formula used for calculating capillary rise of the liquid. 2, g and R are constant, so we are left with surface tension and density of the liquid. Table 1 lists values of surface tension and density of common liquids.

We can calculate the ratio σ/ρ and find a liquid with the highest ratio (and therefore the highest capillary rise).

Table 1 clearly shows that water is the liquid with the highest ratio of σ/ρ. The second highest would be glycerine tree.

Monika Drexlerová
monika.drexlerova@fykos.org

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Table 1: Values of surface tension and density of different liquids.

liquid	σ / $10^{-3}\,\mathrm{N\cdot m^{-1}}$	ρ / $\mathrm{kg\cdot m^{-3}}$	σ/ρ / $10^{-5}\,\mathrm{N\cdot m^2\cdot kg^{-1}}$
acetone	23.3	784	3.0
aniline	40.5	1 020	4.0
benzene	28.9	876	3.3
diethyl ether	16.4	713	2.3
ethanol	22.6	789	2.9
glycerine	62.5	1 260	5.0
chloroform	26.5	1 490	1.8
formic acid	37.8	1 220	3.1
acetic acid	28.0	1 050	2.7
methanol	22.7	792	2.9
n-hexane	18.4	661	2.8
n-pentane	16.0	626	2.6
olive oil	33.0	910	3.6
castor oil	36.4	960	3.8
turpentine oil	27.0	855	3.2
kerosene	27.0	800	3.4
propanol	23.7	803	3.0
mercury	476.0	13 534	3.5
carbon disulfide	33.8	1 260	2.7
tetrachloroethylene	25.9	1 590	1.6
toluene	28.4	867	3.3
water	72.8	1 000	7.3