Problem I.E ... rice with a pun 12 points

Measure the density of grains of raw rice. Do not forget to thoroughly describe your solution and estimate its uncertainty.

Karel was thinking in a canteen in Košice.

Introduction

We measure the density ϱ, which is defined as the ratio of the mass of a body m to its volume V as

$$\varrho = \frac{m}{V}\,.$$

However, we encounter a certain difficulty here – rice is a granular material, so its own volume is not easy to measure directly, because air "pockets" between individual grains significantly increase the total volume. The ratio of mass to total volume (that is, the volume of the rice itself and the space between individual grains) describes a quantity known as *bulk density*[1], which is not the subject of this problem. Bulk density is introduced mainly for practical purposes, where the density itself is less useful information than how much actual rice (or gravel or basalt) can be packed into a given volume.

Measurement method

We could determine the volume of rice directly, that is, by approximating a single grain as an ellipsoid. We would measure its dimensions and then calculate its volume; by weighing it, we would determine its mass and then calculate its density. However, because of the small masses and dimensions of individual grains, even a small inaccuracy that we would most likely make when measuring at home is significant, since it must be related to the absolute measured values, which are small, producing a large relative deviation.

The magnitude of this error would decrease with an increasing number of measurements. To suppress it as much as possible, we could gradually measure a large number of grains in this way, but that would be time-consuming. Another problem is that each grain has a different dimension and shape, so we must average the mass and volume separately. This can also be done cleverly, for example, by arranging the grains in a row and then weighing them, as appeared in one of the participants' solutions.

In our solution, we therefore use a method in which we determine the total volume of the poured rice in a volumetric measuring device (in our case, a graduated cylinder), and then pour in a known volume of liquid that fills the gaps between the individual grains. From the known volume of the liquid and the total volume of the rice with the liquid, we thus obtain the total volume of the rice grains.

However, this method is not suitable for all liquids. Let us start from Archimedes' principle, which describes the behavior of submerged bodies in a liquid. Mathematically, it is formulated as

$$F_{\mathrm{vz}} = V \varrho_{\mathrm{k}} g\,,$$

[1] `https://cs.wikipedia.org/wiki/Hustota`

where F_{vz} is the acting buoyant force, V is the volume of the submerged part of the body, ϱ_{k} is the liquid's density, and g is the gravitational acceleration. The gravitational force F_g also acts on the body, so the resultant force F is

$$F = F_g - F_{\mathrm{vz}} = Vg(\varrho_{\mathrm{t}} - \varrho_{\mathrm{k}})\,,$$

where ϱ_{t} is the average density of the body. From this relation, we see that if we want the rice to sink so that we can measure its entire volume, the resultant force must be positive; that is, we must use a liquid with a lower density than that of rice – which is, after all, a simple and intuitive consideration.

In our solution, we choose water as the liquid, which, as we can see, satisfies this condition. The measurement of rice density is indirect because we calculate the volume of rice V_{r} as

$$V_{\mathrm{r}} = V - V_{\mathrm{v}} = V - m_{\mathrm{v}}/\varrho_{\mathrm{v}}\,,$$

that is, as the difference between the volume of rice with water and the volume of the added water V_{v}. We can find this as the ratio of the water mass to its known density at room temperature, $\varrho_{\mathrm{v}} \doteq 998\,\mathrm{kg{\cdot}m^{-3}}$. Under our conditions, calculating the water volume is more accurate than measuring it in a graduated cylinder, because we have more accurate scales for measuring mass, and the density is easy to find in tables with high precision. Finally, we obtain the density of the grains as

$$\varrho_{\mathrm{r}} = \frac{m_{\mathrm{r}}}{V_{\mathrm{r}}} = \frac{m_{\mathrm{r}}}{V - m_{\mathrm{v}}/\varrho_{\mathrm{v}}}\,,$$

This method also has a disadvantage: if we are not careful and take too long during the measurement, the liquid may soak into the rice. From observation, however, we find that under ordinary conditions, we can use water without significantly degrading the results. We could further improve the measurement by choosing the type of rice. Classic white rice, which is the most common in our region, is polished rice with its husk removed. It is therefore very porous, and the liquid can soak into it more easily. Brown rice could be a better option in this respect.

Measurement procedure

For the measurement itself, we need a scale, a volumetric measuring cylinder, a container in which we weigh the rice and water, and, finally, the rice and water themselves.

In our case, we used a standard laboratory cylinder with a volume of 100 ml and a 1 ml scale, and a ScaleHouse scale with an accuracy of $\pm 0.01\,\mathrm{g}$.

We weigh the rice and water – we convert the water mass to volume using density, because under our conditions, we can measure mass much more accurately than volume. It is good to choose the smallest possible volume of water, but it is absolutely necessary that all the rice be below the surface. We then pour this water into the measuring cylinder with the rice and, after the level stabilizes, read off the total volume.

Results

We repeat the measurement five times and take the density of water to be $\varrho_{\mathrm{v}} = 998\,\mathrm{kg{\cdot}m^{-3}}$. The results are listed in Table 1.

Table 1: Density of rice ϱ

No.	$\frac{m_\mathrm{r}}{\mathrm{g}}$	$\frac{m_\mathrm{v}}{\mathrm{g}}$	$\frac{V}{\mathrm{cm^3}}$	$\frac{\varrho}{\mathrm{g{\cdot}cm^{-3}}}$
1	44.66	63.44	93.0	1.31
2	38.10	62.86	86.5	1.30
3	38.36	60.86	85.5	1.29
4	39.18	62.92	87.0	1.31
5	38.92	60.48	85.0	1.31

Error calculation We estimate the uncertainty using the standard error (SE). It is determined as

$$\sigma_\mathrm{SE} = \frac{s}{\sqrt{N}}\,,$$

where N is the number of measurements and s is the sample standard deviation, determined as

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(\varrho_i - \bar{\varrho})^2}\,,$$

where ϱ_i are the measured densities listed in Table 1 and $\bar{\varrho}$ is the arithmetic mean of these densities.

However, as we have already shown above, our measurement is indirect, and therefore the errors combine. For a difference, absolute errors add, while for a quotient, relative errors add. We determine the error of the measuring instruments using the law of propagation of uncertainty for independent variables in indirect measurements, as

$$\sigma_\mathrm{r} = \varrho\sqrt{\left(\frac{\Delta m_\mathrm{r}}{m_\mathrm{r}}\right)^2 + \left(\frac{\Delta V + \Delta m_\mathrm{v}/\varrho_\mathrm{v}}{V - m_\mathrm{v}/\varrho_\mathrm{v}}\right)^2}\,.$$

The total error is therefore determined as

$$\sigma = \sqrt{\sigma_\mathrm{r}^2 + \sigma_\mathrm{SE}^2}\,.$$

The final result is therefore the value $\varrho_\mathrm{r} = (1.30 \pm 0.01)\,\mathrm{g{\cdot}cm^{-3}}$.

Discussion and conclusion

Although we declared the error caused by liquid absorption to be negligible, it is clear that by using a different measurement method, we could obtain a slight further improvement in accuracy.

The choice of water proved practical because it does not soak in too much. By choosing a suitable liquid, we could suppress this effect further, for example, by using a concentrated salt solution, in which, due to osmotic effects, rice absorbs even less water.

In scientific and industrial practice, methods based on a similar principle are used to measure the density of porous and granular substances. One of these methods is pycnometry. In classical liquid pycnometry, a pycnometer is a glass vessel that has a precise ground joint and a narrow capillary in its stopper. We can fill it with the same, previously known volume every time, which enables very precise measurement. A somewhat different and more interesting method is *gas pycnometry*. A gas pycnometer works based on Boyle–Mariotte's law, which tells us that during an isothermal process in an ideal gas,

$$pV = \mathrm{const}\,.$$

The measurement process itself proceeds as follows: we place our sample into the pycnometer and fill it with an inert gas, which fills the pores of the material. After the measuring chamber is filled with gas, the pressure is measured. The gas is then allowed to expand into a reference chamber whose volume is known in advance. Thanks to the pressure measurement during the expansion of the gas, it is then possible to use the above law to calculate its initial volume, from which we also find the volume of our sample. Helium is often used as a gas because it is not only inert and does not interact with the sample, but is also composed of small atoms, so it more easily enters smaller pores. A cheaper alternative is the use of nitrogen.

In conclusion, we can state that we successfully determined the density of rice by immersion in a less dense liquid. The density of rice was determined to be $\varrho_\mathrm{r} = (1.30 \pm 0.01)\,\mathrm{g{\cdot}cm^{-3}}$.

Maxmilián Ladislav Skuda
maxmilian.skuda@fykos.org

FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of CUNI MFF, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports of the Czech Republic.