Problem I.4 ... falling lens

8 points; průměr 5,52; řešilo 104 studentů

We are holding an object in one hand at a distance D from the ceiling. Where on the vertical axis passing through the object do we have to place a contact lens with a focal length f, so that a focused image is created on the ceiling?

Now, we will place the lens at this distance. The object accidentally falls out of our hand, i.e., it is in free fall. How do we need to move the lens so that the image remains focused? Will the position of the lens approach some specific value after a very long time? Assume that the height of the room is much greater than both D and f. *Adam has dropped his glasses.*

Let us denote a the distance of the falling object from the lens, and a' the distance between the image and the lens. Since we need the image to be focused on the ceiling, the following equation must hold

$$D = a + a' .$$

From the Gaussian lens equation, we know the relationship between a, a', and the focal length f

$$a' = \frac{af}{a-f} .$$

Substituting this into the first equation and simplifying, we get a quadratic equation

$$a^2 - Da + Df = 0 ,$$

which gives us two initial distances a' betweem the lens and the ceiling

$$a' = (D-a) = \frac{D}{2} \pm \frac{\sqrt{D^2 - 4Df}}{2} .$$

For a converging lens (as in this problem), $f > 0$, and for $D > 4f$, we get two possible solutions. For $D = 4f$, the lens must be placed exactly halfway between the object and the ceiling, and for $D < 4f$, such a position cannot be chosen. Therefore, let us consider only the case where $D \geq 4f$ for now.

When the object is dropped, the distance changes according to the relation $d(t) = D + gt^2/2$, so the distance of the lens from the ceiling changes according to the equation

$$a' = \frac{d(t)}{2} \pm \frac{\sqrt{d(t)^2 - 4d(t)f}}{2} = \frac{D + \frac{1}{2}gt^2}{2} \pm \frac{\sqrt{\left(D + \frac{1}{2}gt^2\right)^2 - 4f\left(D + \frac{1}{2}gt^2\right)}}{2} .$$

However, if $D \leq 4f$ is true, a position in which the focused image forms appears as d increases precisely at

$$D + \frac{1}{2} g t_0^2 = 4f \quad \Rightarrow \quad t_0 = \sqrt{\frac{8f - 2D}{g}}\,.$$

The situation stabilizes when $d \gg f$

$$a' = \frac{d}{2} \pm d \frac{\sqrt{1 - 4\frac{f}{d}}}{2} \approx \frac{d}{2} \pm d \frac{1}{2} \left(1 - 4 \frac{f}{2d}\right) = \begin{cases} d - f\,, \\ f\,. \end{cases}$$

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