Problem I.1 ... weight on Europa

3 points; průměr 2,75; řešilo 211 studentů

What would be the gravitational acceleration on the surface of Jupiter's moon Europa if it had the same radius but consisted entirely of liquid water (for simplicity, consider water under normal conditions)? How would this change if it were made entirely of ice? How do the results differ from the actual value? *Karel was thinking about life somewhere else.*

Let's denote the radius of Europa as R and its mass as M. From the basic principle of the gravitational field, the gravitational effect of the entire mass of the moon is equivalent to the situation where all of Europa's mass is concentrated at its center. The gravitational force an object of mass m on the surface of the moon is

$$F = G\frac{Mm}{R^2} = ma\,,$$
$$a = G\frac{M}{R^2}\,,$$

where G is the gravitational constant, and a is the object's acceleration (gravitational acceleration on Europa). If we express the moon's mass as $M = V\rho = (4\pi R^3/3)\rho$ and substitute into the equation for a, we get

$$a = G\frac{M}{R^2} = \frac{4\pi}{3}G\rho R\,.$$

We see that the gravitational acceleration is directly proportional to the density ρ. Europa's radius is 1 560 km and the densities of water and ice are $997\,\mathrm{kg{\cdot}m^{-3}}$ and $920\,\mathrm{kg{\cdot}m^{-3}}$ respectively. We can substitute these values into the acceleration equation, yielding

$$a_{\mathrm{water}} = 0.434\,\mathrm{m{\cdot}s^{-2}} = 0.044g,$$
$$a_{\mathrm{ice}} = 0.401\,\mathrm{m{\cdot}s^{-2}} = 0.041g.$$

When we compare these results with the real value of the acceleration $a_{\mathrm{real}} = 1.31\,\mathrm{m{\cdot}s^{-2}} = 0.134g$, we find that the actual acceleration is nearly three times greater. Therefore, Europa has a higher average density than water under standard conditions. This is because even though its surface is covered in water or ice, Europa is not formed from these compounds only, instead it has a heavy core with significantly higher density. Even if the moon was entirely made of water, we would still need to consider other factors, such as the density of water increasing under higher pressure (and thus at greater depths beneath the surface).

Jonáš Dej
jonas.dej@fykos.org

FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of CUNI MFF, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports of the Czech Republic.