Problem VI.P ... to boil the ocean

10 points; (chybí statistiky)

How long would it take to heat the world's oceans to the boiling point? Consider different energy sources, however, only those that are available on Earth (solar radiation counts).

Michal is fond of the English quantifier "to boil the ocean".

The answer to the question posed in the problem statement seems straightforward: we can simply divide the amount of heat by the power to obtain the time required. Let us analyze how much energy we need, where we can get it, and how we can supply it to the ocean.

The radius of the Earth is $R_Z = 6378 \cdot 10^3\,\mathrm{m}$ and according to the NOAA agency,[1] the average depth of oceans is $\bar{h} = 3688\,\mathrm{m}$, less than one-thousandth of the radius, therefore we can calculate the volume of water simply as a product of the surface times the thickness of this surface layer.[2]

Boiling occurs when the vapor pressure reaches the pressure of the surrounding environment (and the liquid evaporates from its whole volume). Clausius-Clayperon equation describes this phase shift, from which we can derive the relation for the boiling point

$$T_\mathrm{v}(h) = \left(\frac{1}{T_\mathrm{atm}} - \frac{R\ln\left(\frac{p(h)}{p_\mathrm{atm}}\right)}{H_\mathrm{aq}}\right)^{-1} ,$$

where $T_\mathrm{atm} = 373\,\mathrm{K}$ is the known boiling point at $p_\mathrm{atm} = 101\,\mathrm{kPa}$, $p(h) = h\rho g + p_\mathrm{atm}$ is the pressure in depth h and H_aq is the molar heat of boiling. By substituting the values for water found in tables, we find that $T_\mathrm{v}(\bar{h}) \approx 677\,\mathrm{K} = 403\,{}^\circ\mathrm{C}$. This temperature is noteworthy, especially after comparison with the temperature of the critical point of water. After comparing the two, we find that the ocean could not even boil at its average depth. For our following purposes, we will only consider boiling close to the surface but heating in the entire volume to $T_\mathrm{atm} = 373\,\mathrm{K} = 100\,{}^\circ\mathrm{C}$.

The initial temperature of the ocean on the surface is variable, from $-2\,{}^\circ\mathrm{C}$ to roughly $32\,{}^\circ\mathrm{C}$, influenced by the weather or the local climate. This phenomenon manifests itself until reaching the so-called thermocline, from where the temperature does not change and sits between $0\,{}^\circ\mathrm{C}$ and $3\,{}^\circ\mathrm{C}$.[3] The depth of the thermocline is a few hundred meters under the surface. Considering the average depth, let us assume the average initial temperature before heating to be $T_0 = 276\,\mathrm{K} = 3\,{}^\circ\mathrm{C}$.

All things considered, we need[4]

$$Q = 4\pi R_\mathrm{Z}^2 \bar{h} \rho_\mathrm{aq} C_\mathrm{aq} \left(T_\mathrm{v}(0\,\mathrm{m}) - T_0\right) \approx 7.64 \cdot 10^{26}\,\mathrm{J} = 764\,000\,\mathrm{ZJ} = 212\,000\,\mathrm{EWh}$$

of energy for the heating.

Before considering any means for supplying this energy, let us explore the possibilities of covering these energetical requirements. Fossil fuels possess a relatively high energy density. In practice, a distinction is made between combustion heat and calorific value, the latter being lower since it excludes the heat carried away by gaseous water vapor. The higher the hydrogen-to-carbon ratio in the fuel, the greater the difference – around 10% for methane. The combustion heats are listed in Table 1. However, due to the variability of the quality of the fuels and the

[1] https://oceanservice.noaa.gov/facts/oceandepth.html

[2] For simplicity, the following considerations will neglect the one-third surface of the landmass.

[3] https://en.wikipedia.org/wiki/Ocean_temperature

[4] $E = 10^{18}$, $Z = 10^{21}$

Table 1: Energies of the reserves of the fossil fuels

fuel	energy density	reserves	energy
petroleum	$6.1\,\mathrm{GJ/barrel}$[a]	$1.5 \cdot 10^{12}$ barrel[b]	$9.2\,\mathrm{ZJ}$
natural gas	$37.8\,\mathrm{MJ/m^3}$[c]	$2.05 \cdot 10^{5}\,\mathrm{km^3}$[d]	$7.7\,\mathrm{ZJ}$
brown coal	$17\,\mathrm{MJ/kg}$[e]	$410 \cdot 10^{9}$ t[f]	$7.0\,\mathrm{ZJ}$
black coal	$30\,\mathrm{MJ/kg}$[g]	$410 \cdot 10^{9}$ t[h]	$12\,\mathrm{ZJ}$
total	–	–	$36\,\mathrm{ZJ}$

[a] https://en.wikipedia.org/wiki/Barrel_of_oil_equivalent

[b] https://en.wikipedia.org/w/index.php?title=List_of_countries_by_proven_oil_reserves&oldid=1221410516, OPEC 2021 statement

[c] https://en.wikipedia.org/wiki/Heat_of_combustion

[d] https://www.eia.gov/international/data/world/natural-gas/dry-natural-gas-reserves, EIA 2020 statemenet

[e] https://en.wikipedia.org/wiki/Heat_of_combustion#Lower_heating_value

[f] https://en.wikipedia.org/wiki/Coal, BP 2008 statement

[g] https://en.wikipedia.org/wiki/Heat_of_combustion#Lower_heating_value

[h] https://en.wikipedia.org/wiki/Coal, BP 2008 statement

estimates of their necessary amount, this difference is negligible – we do not have to consider the heat transfer from the fuel to the water yet.

The table lists the total reserves of the selected fuels. However, their total energy is still many orders of magnitude lower than the amount of heat required. Therefore, it is unfeasible to boil the ocean using only the known fossil fuels.

Light water nuclear reactor is able[5] to produce $544\,\mathrm{GJ/kg_U}$, Uranium is therefore roughly 10 000 times more energetically dense per weight than petroleum. [6] The same document[7] estimates the economically mineable uranium reserves in the world as $8 \cdot 10^9\,\mathrm{kg_U}$, therefore energy of $4.3\,\mathrm{ZJ}$ is at our disposal. That is even less sufficient for our heating than the smallest of the fossil fuel reserves.

Nuclear decay also occurs naturally in the Earth's crust, providing a potential heat source for heating water. Combined with the heat from the Earth's core, this gives us approximately $\tilde{P}_{geo} = (50 \ldots 100)\,\mathrm{mW/m^2}$ at our disposal.[8] [9] The crust's temperature is the same as the water temperature at the bottom of the ocean. To get the heat into the water, we would have to dig to a depth of about $10\,\mathrm{km}$, where the temperature reaches over $100\,^{\circ}\mathrm{C}$. The transfer of heat by conduction is proportional to the difference of the temperatures and the thermal conductivity λ

$$\tilde{P} = \lambda \frac{\Delta T}{\Delta h}\,,$$

[5] https://www.oecd-nea.org/upload/docs/application/pdf/2020-12/7555_uranium_-_resources_production_and_demand_2020__web.pdf, Appendix 5. Energy conversion factors

[6] We are assuming the use of existing reactors, whereas a complete nuclear decay would theoretically release even more energy. For more details, see the third problem of the second series.

[7] Table 1.2a. Identified recoverable resources, 2019

[8] http://www.withouthotair.com/c16/page_97.shtml, only $10\,\mathrm{mW/m^2}$ from nuclear

[9] https://web.archive.org/web/20110811133919/http://anquetil.colorado.edu/EPP3/readings/Pollack_etal_1993_Rev_Geophys.pdf, only $10\,\mathrm{mW/m^2}$ from nuclear

where $\tilde{P}$ designates the power per unit area. The water has a thermal conductivity of $\lambda_{\mathrm{aq}} = 0.6\,\mathrm{W/m/K}$, therefore, the water column would be able to "pull up" only $\tilde{P}_{\mathrm{aq}} \approx 6\,\mathrm{mW/m^2}$ at this gradient. We would do better by filling the tunnels with copper with $\lambda_{\mathrm{Cu}} \approx 400\,\mathrm{W/m/K}$, which gives us $\tilde{P}_{\mathrm{Cu}} \approx 3.9\,\mathrm{W/m^2}$. The sustainable power from the core is even lower, so the tunnels would, on average, cover $\tilde{P}_{\mathrm{geo}}/\tilde{P}_{\mathrm{Cu}} \approx 2\%$ of the area.[10]

The time necessary to heat the water with such power would be

$$t_{\mathrm{geo}} = \frac{Q}{4\pi R_{\mathrm{Z}}^2 P_{\mathrm{geo}}} = \frac{\bar{h}\rho_{\mathrm{aq}} C_{\mathrm{aq}} \left(T_{\mathrm{v}}(0\,\mathrm{m}) - T_0\right)}{P_{\mathrm{geo}}} \approx 1.6 \cdot 10^6 \text{ years}$$

where we used $\tilde{P}_{\mathrm{geo}} = 30\,\mathrm{mW/m^2}$, because when taking the energy from a greater depth, only the heat from the (Earth's) core remains, and the contribution from the nuclear decay disappears. Considering the age of our planet, geothermal power would be sustainable for a calculated time of t_{geo}.

In analyzing the following sources, we will skip over wind power and hydroelectric power, as in the end, they only use the energy "from the weather", which comes from the sun. Therefore, let us consider the solar energy that we can capture on Earth. With a solar constant $\tilde{P}_{\odot} = 1370\,\mathrm{W/m^2}$, which represents the specific power from the sun at the Earth's distance, we can determine the total value by multiplying it by the Earth's projected area πR_{Z}^2 with respect to the sun

$$t_{\odot} = \frac{Q}{\pi R_{\mathrm{Z}}^2 P_{\odot}} = \frac{4\bar{h}\rho_{\mathrm{aq}} C_{\mathrm{aq}} \left(T_{\mathrm{v}}(0\,\mathrm{m}) - T_0\right)}{P_{\odot}} \approx 138 \text{ years}\,.$$

This result will remind us that we have not yet considered any losses during the heating, because as we know from both atlases and chronicles, the ocean generally does not boil. The heat losses through the bottom are insignificant, as that would "only" expand the question to the heating of the layer of the first 10 kilometers of the Earth's crust, which adjoins the ocean. However, the heated ocean would emit energy from its surface away into space according to the Steffan-Boltzmann law

$$P_{\mathrm{okna}} = 4\pi R_{\mathrm{Z}}^2 \sigma T_{\mathrm{v}}(0\,\mathrm{m})^4 \approx 5.6 \cdot 10^5\,\mathrm{TW}\,,$$

while our most powerful (per wattage) power source – the sun – can only supply

$$P_{\odot} = \pi R_{\mathrm{Z}}^2 \tilde{P}_{\odot} \approx 1.8 \cdot 10^5\,\mathrm{TW}\,,$$

so we would not be able to reach the boiling point this easily.

The emitted energy could be reflected into the water using a giant mirror. However, this would also overshadow the sun, leaving nothing to be reflected. A semi-permeable mirror is not unidirectional; it reflects half the energy and allows the other half to pass through. As a result, we would still lose some of the solar power. Could the radiation be filtered by something other than the direction from which it is coming? The mentioned Stefan-Boltzmann law provides the total flux of energy at a given temperature of the body. Additionally, the temperature affects the radiated spectrum. The wavelength of maximum radiation is inversely proportional to the temperature (derived from Planck's law), a principle known as Wien's displacement law

$$\lambda_{\max} \doteq \frac{hc}{4.97 k_{\mathrm{B}} T}\,.$$

[10] Realization $\tilde{P}_{\mathrm{Cu}}$ would mean replacing 10 kilometers of Earth's crust with copper.

The sunlight has about the same spectrum as a blackbody with a temperature of 5 800 K, while the heated water will have a temperature of $T_\mathrm{v} = 373\,\mathrm{K}$ at most. That corresponds to a different spectrum (the wavelength would be displaced about 15 times).

The sought-after filters are the greenhouse gasses as they let the high-temperature light from the sun pass, but they reflect[11] the low-temperature light from the heated ocean. Whether $P_\mathrm{windows} - P_\mathrm{reflection} \leq P_\odot$ can be reached depends on the convolution (overlay) of the spectra. We leave this task to the dear reader.

Another type of loss that we should evaluate is the water loss from the ocean due to evaporation. Evaporation of a kilogram of water requires about five times more energy than heating the same amount by 100°C ($H_\mathrm{aq}/(100°\mathrm{C} \cdot C_\mathrm{aq})$). That does not mean the water cannot evaporate before it starts to boil. It would take five times longer with the power sources considered previously (if any heating did not occur and the water would only evaporate). But where to put the vapor? The vapor pressure is p_atm at 100 °C. That, according to the equation of state, corresponds to a density of vapor of $\rho_\mathrm{vap} = 0.6\,\mathrm{kg/m^3}$, which we use to estimate the height of a (uniform) water atmosphere

$$h_\mathrm{vap} = \bar{h}\frac{\rho_\mathrm{aq}}{\rho_\mathrm{vap}} \approx 6\,100\,\mathrm{km}\,.$$

This height is not negligible compared to the radius of the Earth, so the assumption that the volume of the atmosphere is simply the product of the surface area and its thickness does not hold. Nevertheless, even with a proper calculation, the thickness of this water atmosphere would be greater than the traditional height of the atmosphere (about 100 km). From that, we can conclude that the whole ocean would not fit in the atmosphere (even less so if the temperature and the vapor pressure were lower). The result would be an equilibrium between the liquid non-vaporized remnant of the ocean and a misty atmosphere above it. [12]

Conclusion

Firstly, we estimated the amount of energy required to boil the ocean. Given the available information, no fuel could meet these energy requirements, so we would need to rely on solar energy (and possibly geothermal energy) instead. If we have completely stopped the energy emission into space, a theoretically lower estimate of the time to boil the ocean would be 138 years. We could implement an imperfect thermal isolation using greenhouse gasses. Even then, a significant portion of the ocean would evaporate (in thousands of years, depending on the

[11] Or rather absorb and disperse in the atmosphere. Thus, the energy would return to the ocean indirectly through the air, rather than directly from reflection.

[12] For those interested, we can highlight *the Messinian salinity crisis*, during which the Mediterranean Sea evaporated. This evaporation was primarily driven by solar energy. Due to the relatively small area, there was enough space for the sea waters in the atmosphere, and therefore, it evaporated before it could start to boil. The same thing happens if we leave a container with water in the sun – if it does not rain, it evaporates.

quality of the greenhouse effect), before it would start to boil.

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