Problem VI.3 ... submarine sickness

5 points; (chybí statistiky)

A submarine of volume $V = 6\,\mathrm{m^3}$*, with solid carbon fiber walls of negligible thickness and internal temperature* $t = 20\,\mathrm{^\circ C}$ *was submerged to the depth* $d = 3\,\mathrm{km}$*. Suddenly, the walls ceased to hold and the submarine shrank. What is the temperature inside?*

Assume that the submarine did not break, but only shrank (although we know from experience that this is not a realistic assumption), and that the passengers and the cargo put up only negligible resistance to the shrinking (this is a realistic assumption). *Darwin Awards 2025*

The submarine is airtight, so the pressure inside is usually approximately equal to the atmospheric pressure $p_\mathrm{a} \doteq 1 \cdot 10^5\,\mathrm{Pa}$, whether it is submerged or not. In this, a submarine differs from, e.g., a diver, who has to adapt to the surrounding pressure. The pressure exerted by water in depth d is atmospheric + hydrostatic $p = p_\mathrm{a} + d\rho g$, so the walls of the submarine are under a massive pressure $d\rho g$, where $\rho \doteq 1 \cdot 10^3\,\mathrm{kg{\cdot}m^{-3}}$ is the density of water, which is practically constant.

Let's consider the thermodynamic process occurring in this scenario. The submarine undergoes a rapid contraction because it offers minimal resistance once the walls give way. Therefore, calculating the temperature changes during this quick process is unnecessary. Instead, we are interested in the temperature at the new point of mechanical equilibrium, where the internal pressure matches the external water pressure p. Reaching thermal equilibrium takes significantly longer, so this rapid process can be approximated as adiabatic.

Next, we shall assume that the walls move significantly slower during shrinking than the speed of sound inside the submarine. That means that during the process, the air inside the submarine always reaches thermodynamic equilibrium – pressure and temperature do not depend on distance from the walls of the submarine. Speed of sound is the "speed of information propagation" in a material. If all volume changes are small compared to this speed, the air inside quickly "exchanges information" about these changes, resulting in uniform pressure and temperature. Hence, we do not need to deal with a time evolution during some complicated non-stationary thermodynamic process but only need to know the initial and final state.

The resulting pressure and volume during an adiabatic process satisfy

$$pV_\mathrm{f}^\kappa = p_\mathrm{a} V^\kappa\,,$$

where V_f is the final volume in mechanical equilibrium, and the adiabatic constant is approx. $\kappa = 1.4$ for air, which fills most of the submarine (we neglect the volume of passengers and cargo). Under the assumption of thermodynamic equilibrium, the equation of ideal gas also holds:

$$p_\mathrm{a} V\ /T = pV_\mathrm{f}/T_\mathrm{f}\,,$$

where $T = 293\,\mathrm{K}$ is the initial and T_f the final absolute temperature. We express V_f from one of the equations, substitute it into the other, and express

$$T_\mathrm{f} = T\left(1 + \frac{d\rho g}{p_\mathrm{a}}\right)^{1-\frac{1}{\kappa}}\,.$$

The final temperature is $T_\mathrm{f} \doteq 1500\,\mathrm{K}$, so approx. $1200\,\mathrm{^\circ C}$.

In the end, we can determine how reality differs from our calculations. After the catastrophe of the Titan submarine, which inspired this problem, several simulations emerged that used specific shapes and materials for the submarine. The pressure resistance was supposed to be provided by a cylinder made of carbon fibers with rounded titanium end caps[1]. The titanium end caps were found whole on the seabed, while the cylinder probably broke into pieces under pressure. Simulations[2] of stresses in these materials and ability to resist small deformations as pressure increases showed that the submarine indeed starts to shrink, especially in the middle of the carbon fiber cylinder, but before volume could significantly decrease, this material shatters. Breaking starts near the cylinder's edge, where it connects with the metal; this is where the material strain is the greatest, causing it to wear down the fastest, leading to the first cracks. The rest of the cylinder then shatters very quickly, filling the submarine with water within milliseconds. The initial deformation is slower, lasting perhaps tens of milliseconds over small distances, supporting our assumption that this process is slower than the speed of sound in air. However, this model only fits a small part of the collapse process. Another claim is that the interior of the submarine became as hot as the surface of the Sun[3], likely based on these types of calculations. However, this claim is only linked to a 3D animation, not a physical simulation. We can also calculate the volume in mechanical equilibrium $V_\mathrm{f}/V = (p/p_\mathrm{a})^{-\frac{1}{\kappa}} \doteq 0.02$; simulations clearly show that the submarine never reaches anywhere near this volume, and therefore not such a high temperature. The rigidity of a carbon fiber composite depends on the manufacturing process, and optimistic estimates suggest it could withstand the pressure involved here. The details regarding the materials used in the Titan submarine are inconsistent and unclear, but it appears that quality was not a priority. The takeaway from this situation is that submarines should be constructed using high-quality materials and designed with the expectation that they will degrade over time!

Jakub Šafin
`xellos@fykos.org`

FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of CUNI MFF, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports of the Czech Republic.

---

[1] with a hole for an acrylic window in one of them; this window probably broke first, but from a physics perspective, such simple weak points are uninteresting and were later omitted in simulations

[2] `https://www.engineering.com/story/a-nonlinear-structural-analysis-of-the-titan-submersible-shows-implosion-and-fracturing`, `https://www.youtube.com/watch?v=2N2cCCeenZk`

[3] `https://www.tiktok.com/@starfieldstudio/video/7247335287162998059`