Problem VI.1 ... ballons with Martin

3 points; (chybí statistiky)

A car is standing on a straight road, with a freely floating helium balloon tied inside. Suddenly, the car starts to accelerate with acceleration $a = 5.0\,\mathrm{km{\cdot}min^{-2}}$. By what angle will the balloon be deflected from the vertical line? What is the direction of the deflection?

Martin would like to hang on a balloon behind a car.

When the car is stationary, several forces act on the balloon. The gravitational force F_G acts downward. The buoyant force caused by helium in the balloon, which is less dense than air, acts upward. Additionally, the tension of the string to which the balloon is attached acts downward. When the car starts accelerating, this system is non-inertial, and the inertial force F_z starts acting on the air molecules. This force is in the opposite direction to the car's acceleration. That causes a pressure difference, with denser air at the rear part of the car compared to the front. The buoyant force F_vz acts in the direction of the pressure decrease. Consequently, it will have a horizontal component, causing its direction to form an angle φ with the vertical. The tension of the string F_T acts in the opposite direction. Besides these forces, the balloon is subjected to the inertial force F_z in the direction opposite to the acceleration of the car and the gravitational force F_G acting downward.

Let's break down the forces in the x direction (horizontal direction) and in the y direction (vertical direction):

$$x:\ F_\mathrm{z} + F_\mathrm{T}\sin\varphi = F_\mathrm{vz}\sin\varphi \quad\Rightarrow\quad (F_\mathrm{vz} - F_\mathrm{T}) = \frac{F_\mathrm{z}}{\sin\varphi}\,,$$
$$y:\ F_\mathrm{G} + F_\mathrm{T}\cos\varphi = F_\mathrm{vz}\cos\varphi\,.$$

From the equation for the x direction, we express $F_\mathrm{vz} - F_\mathrm{T}$ and substitute into the equation for the y direction. From this, we get:

$$\tan\varphi = \frac{F_\mathrm{z}}{F_\mathrm{G}} = \frac{a}{g}\,.$$

We need to convert the units of acceleration to the base units:

$$a = 5\,\mathrm{km{\cdot}min^{-2}} = \frac{5\,000\,\mathrm{m}}{3\,600\,\mathrm{s^2}} \doteq 1.39\,\mathrm{m{\cdot}s^{-2}}\,.$$

The deflection of the balloon is:

$$\varphi \doteq 8.1^\circ\,.$$

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