Problem V.E ... gooey

12 points; průměr 8,63; řešilo 38 studentů

Measure the dependence of a cooking oil's dynamic viscosity η on temperature T. Fit the measured data to function

$$\eta = \eta_0 \exp\left(\frac{T_0}{T}\right) ,$$

and calculate the values of the parameters η_0 and T_0.
Hint: *When fitting the results, plot the horizontal axis as $1/T$. Then, it is possible to fit the data with the required curve even in less advanced software, such as* Excel.

Petr was preparing for laboratory practice.

Theory

A liquid's viscosity can be measured using Stoke's method. It is derived from the fact that during laminar (e.g., non-eddy) flow, the resistance force of the environment applied to a spherical body moving within it is equal to

$$F_{\mathrm{odp}} = 6\pi r\eta v ,$$

where η is the dynamic viscosity, v is the velocity of the body's movement and r is its radius. The laminar flow condition is not strict; it is fulfilled for low values of Reynold's number (Re). Generally, we can consider a flow as laminar if $Re < 2\ 000$. In some literature, a critical value of $Re_K = 20\ 000$ is listed during suitable conditions (as an adequate shape of the tube where the flowing occurs). We can calculate Reynolds number as

$$Re = \frac{2r\rho v}{\eta} = \frac{d\rho v}{\eta} ,$$

where η, r, and v are as above, and ρ is the density of the liquid in which the body moves. Except for the resistance of the environment, gravitational and buoyancy forces act on the sphere. The following equations describe them

$$F_{\mathrm{grav}} = mg ,$$

$$F_{\mathrm{buoy}} = V\rho g = \frac{4}{3}\pi r^3 \rho g ,$$

where V is the volume of the body (from the equation for the volume of a sphere, we can substitute $\frac{4}{3}\pi r^3$), m is its mass, and g is the gravitational acceleration. We will use a value of $g \approx 9.81\,\mathrm{m{\cdot}s^{-2}}$. Because the resistance force is proportional to the velocity, during a freefall, the body accelerates until the gravitational force equals the buoyancy and the resistance force, after which the movement becomes uniform. Then

$$F_{\mathrm{grav}} - F_{\mathrm{buoy}} - F_{\mathrm{odp}} = 0 ,$$

$$mg - \frac{4}{3}\pi r^3 \rho g - 6\pi r\eta v = 0 \tag{1}$$

holds. If we express the body's mass using the formula

$$\rho_t = \frac{m}{V} ,$$

we can express the liquid's viscosity η from the equation (1) as

$$\eta = \frac{2r^2 g(\rho_t - \rho)}{9v} = \frac{d^2 gt(\rho_t - \rho)}{18l}\,,$$

where we have substituted the velocity using the quotient of the path l and the time t and the radius r using half the diameter $\frac{d}{2}$.

For Newtonian liquids, the viscosity is dependent upon temperature according to an empiric relation

$$\eta = \eta_0 \cdot e^{\frac{T_0}{T}}\,,$$

where η_0, T_0 are the parameters which have to be measured. We can do that by measuring the liquid's viscosity at different temperatures and fitting the above-mentioned dependence on our data.

We will measure the body's density using the pycnometric method. We measure the mass of the pycnometer m_1, the mass of the pycnometer with the body in it m_2, the mass of the pycnometer with the body and a liquid of a known density m_3 and the mass m_4 of the pycnometer filled with a liquid of a known density ρ. We get an equation system for the variables V and ρ_t; its solution for the density is

$$\rho_t = \rho \frac{m_2 - m_1}{m_4 - m_3 + m_2 - m_1}\,.$$

The arrangement and the execution of the experiment

We will drop small balls into a graduated cylinder filled with oil and measure the time after which they pass a chosen path. We have chosen a path with a measured length $l = 11.6\,\mathrm{cm}$ in our case; we can estimate the error as half the smallest mark of the gauge, e.g., $\sigma_l = 0.05\,\mathrm{cm}$. We will drop two different types of balls into the graduated cylinder - blue and white ones. We will measure their diameters using an industrial microscope in two perpendicular axes and their density using the pycnometric method described in the theoretical part above. Because the balls are slightly elliptic, we will determine their effective diameter as a geometric average of two diameters measured $d = \sqrt{d_x d_y}$ (thus the diameter the ball would have to have for its cross-section to have the same area as in the case it was perfectly spherical). We can estimate the relative uncertainty of the pycnometric determination of the density as $\eta = 1\%$. The uncertainty of the industrial microscope is $\sigma_d = 0.01\,\mathrm{mm}$, from the formula for the error propagation results, that this projects into the uncertainty of the geometric average as $\sigma_{\overline{d}} = 0.007\,\mathrm{mm}$. We list the values measured in table 1.

Next, for our measurement, we will have to find out the density of sunflower oil at the temperatures we will conduct our measurements. We list these temperatures in table 2; we estimate the density uncertainties as $\eta_\rho = 1\%$.

Now, we can get to measuring the fall time of the balls in the oil. We heat the oil to the appropriate temperature, drop three balls of each color inside, and measure the time necessary for the balls to pass our chosen path. We measured the time with a stopwatch on our phone. Its precission is much greater than our reaction time, which will therefore comprise most of the measurement's uncertainty. We will estimate it as $\sigma_t = 0.2\,\mathrm{s}$. The times measured are listed in the following tables.

Now, we will plot our data into a graph and fit it with the required dependency. Because, in the formula, the temperature does not appear in degrees Celsius, but in Kelvins, we plot a temperature adjusted by the transformation relationship $0\,^\circ\mathrm{C} = 273.15\,\mathrm{K}$.

Table 1: Diameteres and densities of the balls used

ball	ball diameter in x axis	ball diameter in y axis	geometric average of the ball diameter	uncertainty of the geometric average	ball density	density uncertainty
	d_x / mm	d_y / mm	$\overline{d}$ / mm	$\sigma_{\overline{d}}$ / mm	ρ / kg·m^{-3}	σ_ρ / kg·m^{-3}
blue ball	2.14	2.11	2.125	0.007	2593	26
white ball	1.66	1.64	1.650	0.007	2345	23

Table 2: Oil densities at different temperatures

	density at 25 °C	density at 50 °C	density at 65 °C	density at 85 °C	density at 100 °C
	ρ_1 / kg·m^{-3}	ρ_2 / kg·m^{-3}	ρ_3 / kg·m^{-3}	ρ_4 / kg·m^{-3}	ρ_5 / kg·m^{-3}
density	920	900	890	880	870
uncertainty	9	9	9	9	9

Table 3: Measured times and the balls' velocities

teplota	blue ball		white ball	
T / °C	t / s	v / cm·s^{-1}	t / s	v / cm·s^{-1}
24.6	1.72	6.74	2.63	4.41
24.6	1.78	6.52	3.29	3.53
24.6	1.92	6.04	3.35	3.46
52.3	1.21	9.59	1.72	6.74
52.3	1.27	9.13	1.84	6.30
52.3	1.19	9.75	1.85	6.27
65.5	0.87	13.33	1.45	8.00
65.5	1.06	10.94	1.38	8.41
65.5	0.93	12.47	1.20	9.67
85.1	0.68	17.06	1.33	8.72
85.1	0.81	14.32	1.12	10.36
85.1	0.74	15.68	1.32	8.79
105.7	0.66	17.58	1.06	10.94
105.7	0.73	15.89	0.86	13.49
105.7	0.67	17.31	0.93	12.47

From that, we get the required relationship; numerically

$$\eta = 2,7 \cdot 10^{-4} \cdot e^{\frac{1597}{T}} \ .$$

The dependency parameters, including uncertainties (calculated using the propagation of errors formula), are then $\eta_0 = (2.7 \pm 0.9) \cdot 10^{-4}\,\mathrm{kg{\cdot}m^{-1}{\cdot}s^{-1}}$ and $T_0 = (1600 \pm 100)$ K.

Result discussion

The graph shows that viscosity's dependence on temperature is truly exponentially decreasing. Nevertheless, both the graph and the resulting uncertainties of the parameters show that our measurement was not the most precise. There are quite a few reasons. First of all, how well was the laminar flow condition fulfilled. Generally, a condition $Re < 2\ 000$ is accepted. However, this is often significantly restricted to $Re \ll 1$ for precise measurements. The relation for the resistance of a sphere in a laminar flow $F = 6\pi r\eta v$ is derived by linearization of the real resistance at low values of Reynolds number, and the lower the number, the better it holds. Even at the first measured temperature, the flow during the fall of the blue ball had a Reynolds number $Re = 2.12$, and during the fall of the white ball $Re = 0.96$, it only increased as the temperature did.

Another reason is the short path on which we have measured the velocities of the balls. Because the balls could pass it in a very short time (comparable to our reaction time), the relative uncertainty of the measurements was very high. We could fix that by measuring on a much longer path.

We have considered the movement uniform rectilinear. However, the resistance of the oil is dependent on the velocity of the ball. Therefore, the ball asymptotically accelerates to a certain critical value. Because of that, we did not measure the path right from the surface of the oil, but we left a few centimeters for a "runway", after passing which the ball's movement could be considered uniform rectilinear in a good approximation.

All oils of the same kind do not necessarily have the same physical properties; they can significantly differ depending on the particular type of processing, for example. From that, it follows that the found densities at different temperatures will not describe our particular oil exactly. Additionally, we did not manage to heat the oil exactly to the temperature respective to the given density.

We have neglected the influence of the graduated cylinder's finite size. Our theory corresponds to a situation in which the liquid was not enclosed in a container of a finite size. If we wanted to be more precise (which did not make much sense due to the precision of our measurement), we could have added a correction term to the equation for calculating viscosity (see the experimental problem in the 6th series of the 33rd year).

Conclusion

We have measured the dependence of the dynamic viscosity of sunflower oil on temperature. In concordance with the theory, the dependence was exponentially decreasing according to the relation $\eta = \eta_0 \cdot e^{(T_0/T)}$.

The parameters of the temperature dependence were determined as $\eta_0 = (2.7 \pm 0.9){\cdot}10^{-4}\,\mathrm{kg{\cdot}m^{-1}{\cdot}s}$ $T_0 = (1597 \pm 108)$ K. The uncertainties of the parameters are significant; our measurement was therefore quite imprecise, especially due to a big relative uncertainty of the time measurement and inaccuracy in determining the oil's density. For greater precision, we could use a viscosimeter intended for such a purpose

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