Problem V.3 ... bowling

6 points; průměr 4,38; řešilo 60 studentů

Jirka was bowling with his friends. He was throwing the ball so that when it hit the lane it had a horizontal velocity v_0 and glided on the lane without spinning. However, there was a coefficient of friction f between the lane and the ball, hence after time t^* the ball started to roll without slipping. Determine the final velocity v^* at this equilibrium, the time t^* and the distance s^* the ball travels before reaching the equilibrium. The ball is solid, with radius r and mass m.

Jirka didn't trust the lecturer, so he made up his own problem.

The ball's movement on the bowling lane is affected by the friction T from the pad. Interestingly, this force appears only when a point in contact with the pad is moving or when some other force is trying to set the point in motion. In either case, it acts against the direction of motion, and its maximum possible magnitude is

$$T = F_\mathrm{N} f \,,$$

where F_N is the normal force. In our case, the frictional force only acts if the sphere is slipping on the surface. Otherwise, the point on the ball in contact with the pad does not move, and there is no other force on the ball in the direction of motion. As long as the sphere is slipping, we have from Newton's second law (or from the impulse-momentum theorem, if you prefer)

$$T = ma \,.$$

The frictional force also generates torque, given by $M = Tr$. From Newton's second law of rotation, we get

$$Tr = J\varepsilon \,,$$

where $J = \frac{2}{5}mr^2$ is the moment of inertia of the sphere and ε is the angular acceleration of the sphere. Thus, we have a system of two equations, and by eliminating the frictional force, we get

$$a = \frac{2}{5} r\varepsilon \,.$$

We know that $v \to v^*$ and $\omega \to \omega^*$, while for a ball that rolls without slipping, there is a formula linking the velocity and the angular velocity of the sphere

$$\omega^* = \frac{v^*}{r} \,.$$

The equations of motion show that the acceleration and angular acceleration are constant (as force and torque are invariable). Then, we can calculate the final velocities v^* and ω^* as

$$v^* = v_0 - at^*$$
$$\omega^* = \varepsilon t^* \,,$$

where t^* marks the time to reach equilibrium. By putting $\omega^* = v^*/r$ and $a = \frac{2}{5}r\varepsilon$, we get

$$\frac{2}{5} v^* = at^* \,,$$

and then

$$v^* = \frac{5}{7} v_0 \,.$$

We have observed that the friction is equal to $T = mgf$ while $v \neq \omega r$ is valid. For the acceleration, we have

$$a = gf\,.$$

We can calculate time t^* simply as the time required to change the velocity from v_0 to v^*, hence

$$t^* = \frac{v_0 - v^*}{a} = \frac{2v_0}{7gf}\,.$$

For a distance of uniformly decelerated motion with initial velocity v_0 holds the following

$$s^* = v_0 t^* - \frac{1}{2}a(t^*)^2\,,$$

from where, by substituting the previous results, we get

$$s^* = \frac{12}{49}\frac{v_0^2}{gf}\,.$$

We could also calculate the distance from the work done by the friction. This work has been done to overcome friction and to spin the sphere and is given by the difference of the kinetic energies of the sphere at the beginning and after reaching the equilibrium state

$$W = Fs^* = \frac{1}{2}mv_0^2 - \frac{1}{2}m{v^*}^2 = \frac{1}{2}\frac{49-25}{49}mv_0^2 \quad\Rightarrow\quad s^* = \frac{12}{49}\frac{v_0^2}{gf}\,.$$

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