Problem V.2 ... basic problem of acoustics

3 points; průměr 1,79; řešilo 78 studentů

Adam can take meaningful notes at the speed v_1. Unfortunately, his calculus professor speaks at the speed of v_2. There is an airflow in the lecture hall, moving from Adam towards the professor, with the air flowing at a velocity of v_3. At what velocity and in which direction along a straight line intersecting Adam and the lecturer should Adam move to transcribe everything the lecturer says into his notebook? Adam likes the word "meaningful".

We will rewrite the speed of Adam's writing v_1, and the speed of the lecturer's speaking v_2, in terms of frequencies. The rate of production (notation) of words u is actually the number of words N produced (noted) over time T. Therefore $u = N/T = Nf$, where f is the frequency of production. Thus, the difference between v_1, v_2 and f_1, f_2 is in the multiplication by the constant N.

To ensure Adam has time to write his notes, he cleverly uses the Doppler phenomenon. When he moves in the direction away from the speaker at a speed of v, he perceives the speaker's voice at a lower frequency

$$f_2' = f_2 \frac{(c - v_3) - v}{c - v_3},$$

where c is the speed of sound in the air in the room. We will find the velocity v for which the frequency f_2' is equal to the frequency f_1

$$\begin{aligned} f_1 &= f_2 \frac{(c - v_3) - v}{c - v_3}\,, \\ v_1 &= v_2 \frac{(c - v_3) - v}{c - v_3}\,, \\ \frac{v_1}{v_2}(c - v_3) &= (c - v_3) - v\,, \\ v &= (c - v_3)\left(1 - \frac{v_1}{v_2}\right)\,. \end{aligned}$$

Thus, if Adam wants to write everything down, he has to run away from the lecturer at the speed of $v \geq (c - v_3)\,(1 - v_1/v_2)$.

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