Problem IV.P ... efficient lighting

10 points; průměr 6,00; řešilo 52 studentů

Describe the basic physical principles of the various methods of producing artificial lighting. Calculate the efficiency for at least three of them, i.e. how much energy supplied is actually converted into visible light. Compare with actual data.

Jarda was replacing his grandmother's lamp switch.

Photometric quantities

Several different quantities with their units are used to determine how much light a given light source emits. We call these quantities photometric since they only deal with visible light. Even if the source emits a significant amount of infrared or ultraviolet light, the values of photometric quantities can be low.

For our purposes, the most important photometric quantity is the *luminous intensity* and its unit *candela*, which tells us how much visible light is emitted by the light source. Candela is one of the SI base units, and historically, it was defined as the amount of light emitted by a candle of given parameters. However, since 2018, it has been defined as "the luminous intensity of a light source that in a given direction emits monochromatic radiation with a frequency of 540 THz and whose radiant intensity in that direction is 1/683 watts per steradian". We can note that the definition contains several constants, which we generally try to avoid. The frequency 540 THz is a yellow light and originates from the fact that the human eye is most sensitive to this frequency. The power 1/683 W is included to ensure that the new definition is consistent with the previous one.

We are also interested in the unit of *luminous flux*, *lumen*, which is derived from candela. In contrast to candela, the lumen takes into account the total visible light emitted by the light source. An important characteristic of the source is the ratio of luminous flux to input power. The higher the ratio, the more efficient the light source can be considered.

Lux is the unit of the quantity E_{v}, which is called *illuminance*. This photometric quantity is defined as the ratio of *luminous flux* Φ_{v} to the area through which the flux passes. Therefore, the illuminance has a value at each point in space.

Black body

(Tungsten) light bulb The light bulb glows because the current flowing through the tungsten filament heats it to a high temperature – typically 2 500 K to 3 300 K. Tungsten is used due to its high melting point. Every physical object at temperature T emits thermal electromagnetic radiation. However, the total emitted power, as well as the spectral region in which the maximum of the emission is located, depends on the temperature. The relationship between the wavelength at which the black body emits most intensely and its thermodynamic temperature is called Wien's displacement law and has a simple form

$$\lambda_{\max} = \frac{b}{T} = \frac{2.898\,\mathrm{mm{\cdot}K}}{T}\,.$$

To get the maximum of the emission in the visible light region, we need the temperature to be definitely greater than 1 000 K. In the case where the maximum would lie well outside the visible light region of the spectrum, the efficiency of the source (in the sense of what input power is needed per luminous intensity) would be very low. The temperature of the solar photosphere is around 5 800 K, which corresponds to a wavelength of $\lambda_{\mathrm{S,max}} = 500\,\mathrm{nm}$ in the yellow-green color region.

Until now, we have only discussed the wavelength at which a given body emits the most. However, the black body emits at all wavelengths λ, and the intensity dependence on wavelength can be expressed using Planck's law

$$u(\lambda, T) = \frac{8\pi hc}{\lambda^5 \cdot \left(\exp\left(\frac{hc}{\lambda kT}\right) - 1\right)}\,,$$

alternatively, in terms of frequency

$$u(f, T) = \frac{8\pi hf^3}{c^3} \frac{1}{\exp\left(\frac{hf}{kT}\right) - 1}\,.$$

Our goal is to determine how much energy is emitted in the visible light region – between 400 nm and 800 nm. We can do that easily by integrating Planck's law within the mentioned bounds. We can simplify the form of the integral mathematically by making the substitution $x = hc/(\lambda kT)$. Substituting for λ in the rest of the integrand and replacing the differential $\mathrm{d}\lambda$ using the following equation

$$x = \frac{hc}{\lambda kT} \quad \Rightarrow \quad \mathrm{d}x = -\frac{hc}{kT}\frac{1}{\lambda^2}\,\mathrm{d}\lambda \quad \Rightarrow \quad -\lambda^2 \frac{kT}{hc}\,\mathrm{d}x = \mathrm{d}\lambda\,,$$

the integral of the law takes the form

$$\int u(\lambda, T)\,\mathrm{d}\lambda = \int \frac{8\pi hc}{\lambda^5 \cdot \left(\exp\left(\frac{hc}{\lambda kT}\right) - 1\right)}\,\mathrm{d}\lambda = -\frac{8\pi k^4}{h^3 c^3} T^4 \int \frac{x^3}{\mathrm{e}^x - 1}\,\mathrm{d}x\,.$$

We have performed a so-called dimensionless substitution, where the exponent is replaced by one dimensionless variable, and the entire expression is adjusted accordingly. The advantage of this step lies not only in the simpler form of the integrand but, more importantly, it allows us to see the dependencies on other physical quantities, in this case, the temperature. The only downside is the need to recalculate the bounds of integration; however, their value will no longer depend on a used system of units since even the bounds are now dimensionless.

Since we are interested in efficiency, we only need to put the integrals on the chosen bounds into a ratio, according to the previous paragraph. For the visible light region, we substitute

$$x_1 = \frac{hc}{kT}\frac{1}{400\,\mathrm{nm}} = 13.35\,, \qquad x_2 = \frac{hc}{kT}\frac{1}{800\,\mathrm{nm}} = 6.67\,,$$

and for all wavelengths we integrate from ∞ to 0. Here, the emitted power is proportional to the fourth power of the temperature, which we chose to be 2 700 K.

By substituing we get

$$\eta = \frac{-\int_{x_1}^{x_2} \frac{x^3}{\mathrm{e}^x - 1}\,\mathrm{d}x}{-\int_{\infty}^{0} \frac{x^3}{\mathrm{e}^x - 1}\,\mathrm{d}x} = \frac{0.600}{\frac{\pi^4}{15}} = 0.092\,.$$

Interestingly, the integral in the denominator can be calculated analytically; however, for the one in the numerator, one of the online numerical calculators, such as WolframAlpha, needs to be used.

If we put the temperature of the Sun, 5 800 K, to the bounds of integration, the efficiency is about five times higher. Therefore, the Sun emits roughly 50 % of its radiant power in the form of visible light.[1] Unfortunately, we do not have suitable material available that could withstand such high temperatures to emit with such efficiency.

We also need to compensate for the fact that the human eye perceives different wavelengths with varying sensitivity. We do this by multiplying proper integrand by tabulated values (denoted by $c(\lambda)$)[2] for the eye's sensitivity. The ratio between these values is the desired efficiency

$$\eta = \frac{\int_{400\,\mathrm{nm}}^{800\,\mathrm{nm}} c(\lambda)\, u(\lambda, 2\,700\,\mathrm{K})\, d\lambda}{\int_0^\infty u(\lambda, 2\,700\,\mathrm{K})\, d\lambda} = 0.019\,,$$

which corresponds to the tabulated[3] range from 1 % to 2 %.

Let us add that in light bulbs, inert gases at low pressures are typically used, ensuring low heat transfer by conduction and convection between the filament and the walls of the bulb. If the bulb were filled with, for example, oxygen, it would lead to the creation of tungsten oxide and its rapid evaporation, reducing the light bulb's lifespan dramatically. In the next section, however, we will learn that the efficiency can be significantly improved by changing the filling of the bulb.

Halogen lamp As we have seen in the previous section, increasing the temperature closer to the temperature of the Sun significantly increases the proportion of radiation in the visible range of the spectrum. The halogen lamp operates on the same principle as the tungsten light bulb, the difference being that a halogen is added as the gas filling of the bulb. This prevents the tungsten from evaporating from the surface of the filament, thus prolonging the lamp's lifespan. Thanks to the special filling, significantly higher temperatures can be achieved, which, as we have seen earlier, leads to an increase in the luminous intensity and efficiency of the bulb. In the calculation, we use the same formula as for the tungsten bulb, but we change the temperature to 3 500 K

$$\eta = \frac{-\int_{10.30}^{5.15} \frac{x^3}{\mathrm{e}^x - 1}\,\mathrm{d}x}{-\int_{\infty}^{0} \frac{x^3}{\mathrm{e}^x - 1}\,\mathrm{d}x} = \frac{1.42}{\frac{\pi^4}{15}} = 0.22\,.$$

After accounting for the eye sensitivity, we get

$$\eta = \frac{\int_{400\,\mathrm{nm}}^{800\,\mathrm{nm}} c(\lambda)\, u(\lambda, 3500\,\mathrm{K})\, d\lambda}{\int_0^\infty u(\lambda, 3500\,\mathrm{K})\, d\lambda} = 0.057\,.$$

We can notice that there is still quite a significant difference between these two efficiencies. This difference is caused by the fact that the human eye sees the green color best and red and violet worst. For a halogen lamp, the maximum of the emission is near the boundary of the visible light ($\approx 830\,\mathrm{nm}$). Therefore, it emits a lot of visible light, but a large portion of it is red, which is less processable for the eye. On the other hand, the halogen lamp is a significant improvement over the classical tungsten bulb.

[1] We could say that it is a relatively high efficiency. However, we must realize that the Sun was here before the human eye was. Thus, the eye evolved to be able to analyze the components of the light in the interval where the radiation intensity is still relatively high. Therefore, the visible light range is determined by the highest intensity of solar radiation, not the other way around.

[2] `https://web.archive.org/web/20070927222337/http://www.cvrl.org/database/text/lum/ssvl2.htm`

[3] We get the tabulated values from `https://en.wikipedia.org/wiki/Luminous_efficacy`.

Candle

A candle is interesting because it has been used as a standard illumination measurement for a long time. Over time, many different "candles" have been used for this purpose, but they all operate on the same principle – they have some fuel that burns. This burning generates heat and light.

The total heat of combustion (how much energy is stored in the fuel) can be found for each fuel. For our case, let us choose a paraffin candle, where the net heat of combustion released by burning paraffin is $E_{\mathrm{par}} = 43.1\,\mathrm{MJ{\cdot}kg^{-1}}$. Additionally, we will determine how long the fuel can sustain the flame, i.e., how long the candle burns. We have to find the burning rate on the internet[4], where we find that the candle burns at a rate of $\dot{m} = 6.3\,\mathrm{g{\cdot}h^{-1}}$.

From there, we can determine the efficiency. Firstly, we calculate the energy consumed per second, i.e., the input power

$$P = E_{\mathrm{par}} \cdot \dot{m} \doteq 75\,\mathrm{W}\,.$$

We know that our standard candle has a luminous intensity of $I = 1\,\mathrm{cd}$. If we consider it, for simplicity, as a spherically symmetric omnidirectional source, we can determine its total luminous flux. Assuming the candle emits light equally in all directions, we multiply the luminous intensity by the number of steradians in a sphere (4π sterad), giving us a total luminous flux of $P' = 12.56\,\mathrm{lm} = 0.0184\,\mathrm{W}$.[5] Now we can easily calculate the efficiency as a ratio of output power to input power

$$\eta = \frac{P'}{P} = \frac{0.018\,4\,\mathrm{W}}{75\,\mathrm{W}} = 0.024\,\%\,.$$

In terms of efficiency, we are two orders of magnitude lower compared to light bulbs. Therefore, we see that candles are not a very good light source. This is because the combustion produces mainly heat, which is not captured in any way by the candle, and it freely dissipates into the surroundings. The processes around the wick are more complex than those in the classical bulbs since there is no controlled, exhausted atmosphere, and many chemical reactions occur. Access to oxygen is crucial for the candle burning (fuel oxidation). In the vicinity of the hot wick, gas molecules even ionize. Because of that, we have mainly focused on the experimental determination of efficiency.

Transition between energy levels of the atomic shell

Fluorescent lamp Fluorescent lamps use low-pressure mercury vapors to create a glow discharge that produces UV light. When switched on, the gas inside is ionized, creating a plasma that conducts electrons and charged particles between the electrodes at the ends of the lamp. The directed motion of the particles causes collisions, leading to excitations and ionizations of electrons in atomic shells. Radiation is emitted upon reentry to the ground state. Depending on the energies, radiation is emitted at the appropriate wavelengths. Another source of radiation is so-called bremsstrahlung, which is produced when the speed of the electrons in the lamp changes.

Additional electronics are required to ignite the glow discharge because the number of charged particles in the gas volume is too low when the lamp is switched off. However, when a certain voltage is reached, avalanche ionization can occur, which allows the discharge to sustain itself even without the starting electronics.

[4] `https://tsapps.nist.gov/publication/get_pdf.cfm?pub_id=101159`
[5] `http://hyperphysics.phy-astr.gsu.edu/hbase/vision/lumpow.html`

The UV light that is created in the gas volume then strikes the surface of the fluorescent lamp, which is covered with a layer of phosphor. The phosphor absorbs the radiation and then emits it in the form of white visible light. Other fillings are also used instead of mercury; for example, neon (or another inert gas) in conventional neon banners or sodium, which was used as the cheapest light source for street lighting. Sodium shines directly in yellow light, so there is no need for a phosphor on the inner surfaces of the lamps. Nowadays, the fluorescent lamps are gradually being replaced by LEDs.

Arc discharge In addition to the glow discharge in fluorescent lamps, we can also encounter the use of plasma for lighting sources that work with arc discharge. Here, the distance between the electrodes is shorter, the voltage between them is lower, but the electric current passing through is much higher, and the temperature reaches several thousand kelvins (which is why arc discharge can be used for welding metals). It was formerly used for street lighting, and the Czech inventor František Křižík contributed significantly to the improvement of these lamps at the end of the 19th century. Today, the arc discharge is used as a light source, for example, in cinema projectors.

LED LED operates on the principle of a diode, where two different types of semiconductors meet. In the first approximation, an N-type semiconductor has the valence band on a higher energy level than a P-type semiconductor. When an electron passes from an N-type to a P-type layer, it loses part of its energy. This energy is emitted in the form of radiation. The value of the emitted energy is determined by the materials of both types of semiconductors; a particular diode only ever shines in a narrow range of wavelengths. Therefore, several different diodes of different wavelengths are needed to create a so-called white light.

We will not concern ourselves with their efficiencies since there are large differences between different dopants, and much depends on the voltage, the temperature of the surroundings, and other external conditions. An interesting fact is that experimental LEDs exist that operate with an electrical efficiency higher than 100 %.[6]

Other light sources

There are, of course, other light sources and principles of light creation (such as laser, neon lamp, cathodoluminescence, etc.). However, their use as light sources is limited in our everyday lives, so we have decided to leave them out of our solution.

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[6] https://phys.org/news/2012-03-efficiency.html