Problem IV.2 ... they got off in Hněvice

3 points; průměr 2,45; řešilo 75 studentů

Tomáš got into the train wagon in the shape of a rectangular cuboid and decided to take a nap. When he woke up, he found that he was alone in the wagon, which was suspended at its geometric center on a cargo crane and rotating around the hinge axis at an angular velocity of ω. Tomáš didn't notice it at first since he was sitting in the wagon's centre with a width of d. When he realized it, he was pleased because he thought of using one of his kilogram standards, which he carries around for situations like this, to measure the length of the carriage. After a few attempts, he managed to throw the standard at an initial velocity of $\vec{v}$ so that after two revolutions of the wagon, the standard hit the far corner of the wagon and broke the window. Neglecting air resistance, what length L of the wagon did he determine?

Tomáš fell asleep on the train and was thrown off by the conductor.

We will look at the whole problem from the point of view of an inertial frame of reference connected to the ground with the origin in the center of the wagon, where Thomas is standing, and we will describe the perceived motion of the corner of the wagon and the etalon.

The corner moves uniformly on a circle with a radius r that is also half the diagonal of the wagon and with an angular velocity of ω. The etalon moves uniformly in a straight line in the horizontal direction with velocity v and accelerates in the vertical direction with gravitational acceleration g. It will be sufficient for us to solve for the motion of the etalon in the horizontal direction since the corner of the wagon does not move in the vertical direction. We know that the wagon managed to rotate twice before the collision occurred, so for the impact time T thus, we have

$$\omega T = 4\pi \quad \Rightarrow \quad T = \frac{4\pi}{\omega} \,.$$

At the time of impact T the etalon and the edge corner must be at the same place, so in the time T the etalon must travel in the horizontal direction just the distance r. This gives us the desired length of the wagon L

$$\begin{aligned} v\frac{4\pi}{\omega} &= \sqrt{\left(\frac{d}{2}\right)^2 + \left(\frac{L}{2}\right)^2} \,, \\ 4\left(v\frac{4\pi}{\omega}\right)^2 &= d^2 + L^2 \,, \\ L &= \sqrt{\left(\frac{8\pi v}{\omega}\right)^2 - d^2} \,. \end{aligned}$$

Addition to the solution The solution above is correct only when assuming a horizontal throw, which we did when writing the solution. However, we did not realize that we had not stated that anywhere explicitly. If we considered a general projectile motion, we would have to state the angle α between the vector of the initial velocity $\vec{v}$ and the vertical axis. The solution would look the same, except instead of the magnitude of the velocity v, we would have to work with the projection of the velocity vector into the horizontal plane, which would be equal to $v \sin \alpha$.

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FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Media Communications and PR Office and is supported by Institute of Theoretical Physics of CUNI MFF, his employees and The Union of Czech Mathematicians and Physicists. The realization of this project was supported by Ministry of Education, Youth and Sports of the Czech Republic.