Problem I.2 ... train shifting

3 points; průměr 2,77; řešilo 230 studentů

Jarda is standing at the end of the platform, waiting for his train to arrive. When the train's first carriage passes him, he discovers that this is the carriage where he has his seat ticket. At this point, the speed of the train is $8.5\,\mathrm{m\cdot s^{-1}}$*, and the train begins to slow down steadily until it stops in* $28\,\mathrm{s}$*. Jarda immediately starts walking in the direction of his carriage, but because he has to push through the crowds of passengers, his speed is only* $1\,\mathrm{m\cdot s^{-1}}$*. What is the shortest time the train must stay in the station for Jarda to board his carriage?*

Jarda is going to Prague again.

The train began to slow down steadily the moment Jarda saw the number on the first carriage. It came to a complete stop in time T, during which it traveled the braking distance s. Thus the first car, in which Jarda has his seat, has then traveled the distance s, and therefore Jarda must also walk the distance s. The problem can be solved in two different ways: using distances or using times.

The total distance s can be divided into two parts: the distance s_1, which Jarda covers while the train is slowing down, and the distance s_2, which Jarda covers while the train is standstill, where the following applies

$$s = s_1 + s_2\,.$$

The train decelerates uniformly, so the braking distance can be expressed as

$$s = \frac{1}{2}aT^2 \quad \Rightarrow \quad s = \frac{1}{2}\Delta v T\,.$$

where a is the deceleration, and $\Delta v = v - v' = v$ is the total change in velocity given as the difference between the train's initial velocity $v = 8.5\,\mathrm{m\cdot s^{-1}}$ and its final velocity $v' = 0\,\mathrm{m\cdot s^{-1}}$. By substituting into the relation for the breaking distance and expressing the time the train is standstill in the station $t_{\mathrm{stationary}}$, we get

$$\frac{1}{2}vT = uT + ut_{\mathrm{stationary}} \quad \Rightarrow \quad t_{\mathrm{stationary}} = T\left(\frac{v}{2u} - 1\right) = 91\,\mathrm{s}\,,$$

where u is Jarda's velocity.

Let's now solve the problem using times. For the train to be in the station for as short as possible, the braking time T and the stationary time $t_{\mathrm{stationary}}$ must be equal to the total time t_{Jarda} it takes Jarda to get to his carriage.

$$t_{\mathrm{Jarda}} = T + t_{\mathrm{stationary}} \quad \Rightarrow \quad t_{\mathrm{stationary}} = t_{\mathrm{Jarda}} - T\,.$$

The time it takes Jarda to walk the braking distance of the train is

$$t_{\mathrm{Jarda}} = \frac{s}{u} = \frac{vT}{2u}\,.$$

By plugging into the relation for $t_{\mathrm{stationary}}$ we obtain

$$t_{\mathrm{stationary}} = \frac{vT}{2u} - T = T\left(\frac{v}{2u} - 1\right) = 91\,\mathrm{s}\,.$$

In both cases, we obtained a result of 91 seconds, which is the shortest time the train must stay in the station for Jarda to board his carriage.

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