

Serial: *Luminous Intensity and Others*

In the last installment of this year's series, we will delve into a field of physics that we encounter only rarely in school and in solving FYKOS problems – photometry. Photometric quantities describe the perception of light by the human eye. In most physical applications, quantities given by energy, power, or radiation flux (with corresponding units defined by the combination of the kilogram, meter, and second) – so-called radiometric quantities – are used to describe radiation. However, the perception of light does not depend simply and straightforwardly on these quantities. The topic of describing a light source developed long before it was possible to measure the energy of radiation.

Finally, in brief appendices, we will mention that the definition of quantities is not the only thing the SI system specifies. We will also look at several other unit systems that are used with advantages in some specific fields.

Candela

Exploring Light

Since antiquity, the fields of astronomy, optics, photometry, and vision theory have developed simultaneously. One of the first records of numerical values associated with a measure of light was the classification of stars into brightness classes by Hipparchus of Nicaea in the second century BCE. In the second century, Ptolemy expanded on his work in the *Almagest*, providing indications of stellar brightness, known as magnitude, for over a thousand stars. The brightness of stars was determined through visual comparison of the sky, a method employed even after the invention of the telescope until the mid-19th century.

The nature of light and vision developed simultaneously – in his work *Optics*, Euclid describes vision using a set of straight rays emanating from the eye, which he uses to describe visual perception and resolution. Many Arab scholars in the early Middle Ages followed his work. However, Alhazen argued against it in the *Book of Optics* at the beginning of the 11th century. Bodies emitting their own light create light rays that illuminate other bodies. Secondary light rays originating from them then enter the human eye. He further describes transparency and color of bodies. Significant new findings came with the development of science in the 17th century. In 1604, Johannes Kepler published the work *Astronomiae Pars Optica* (literally *The Optical Part of Astronomy*), in which he describes the dependence of illuminance as inversely proportional to the square of the distance from the source. In 1704, Isaac Newton published *Optics*, in which he describes, among other things, the propagation of light and the dispersion of light by a prism. Thus, he explains the color of light and objects around us. Subsequently, in 1729, Pierre Bouguer described the absorption of light by the atmosphere depending on the height above the horizon. He is also credited with the first quantitative photometric measurement, determining that the illuminance by the Sun is 300 000 times stronger than by the Moon.

The term photometry, for the discipline describing the measurement of quantities of light, was created by Johann Heinrich Lambert in 1760 with the publication of the book *Photometria*. In it, he demonstrated the following:

- illuminance is inversely proportional to the square of the distance from the source,
- illuminance is proportional to the cosine of the angle between the perpendicular to the incident surface and the direction to the source of radiation,
- light is attenuated exponentially when passing through an absorbing medium – Lambert-Beer law.

His work was experimental using relatively simple tools – candles, screens, mirrors, colored glasses, etc. He was one of the very first to quantify measurement errors. He also measured the reflectivity of different surfaces, observed the change in pupil size with different illumination from its surroundings, described twilight, additive mixing of colors, and contributed to the birth of quantitative astrophysics. He postulated that a radiation source emits light in various directions also proportionally to the cosine of the angle between the direction of radiation and the perpendicular to the surface. Such a source is called Lambertian. He also defined a perfectly diffuse surface where dispersed light has the same cosine dependence regardless of the angle of incidence – so-called Lambertian scattering.

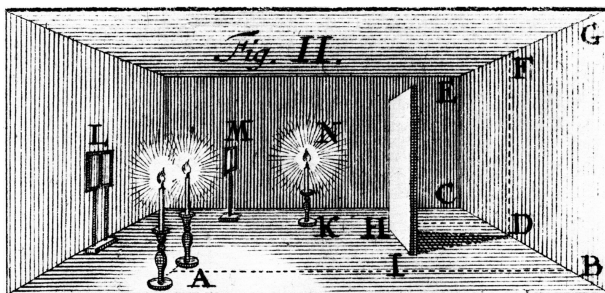


Figure 1: Image from the book *Photometria* by J.H. Lambert, showing the experimental setup with which he derived the basic relationships of photometry¹

Visual Photometer

All historical methods of measuring luminous intensity are based on visually comparing the illumination at two different places. The human eye can detect a difference in illuminance of two surfaces of about 1 %, and this relative measurement accuracy remains the same over a wide range of values of illuminance. Similar to hearing or other sensory perceptions, the observed response is linear in the logarithm of the original quantity – so-called Weber’s law. Because it is a comparative measurement, we require not only the detector (the experimenter’s eye) but also a standard light source of known intensity, referred to as a standard candle. The luminous intensity of other sources was then measured in multiples of this standard. This standard was different in each country. In Great Britain, they used a candle made from the waxy material from the cranial cavity of sperm whales weighing 76 grams and burning at a rate of 7.8 grams per hour. In France, they used the Carcel lamp burning rapeseed oil. In Germanic countries (politically speaking, also in the Czech Republic and Slovakia), the Hefner lamp burning amyl

¹<https://en.wikipedia.org/wiki/Photometria>

acetate (pentyl acetate) with a 40 mm high flame was used as the standard. A single common international standard began to be used only in the mid-20th century.

But how did the actual measurement take place? In a darkened room with black, light-absorbing walls and equipment, the measured and standard light sources were placed along with a photometer – a device used for comparison. By changing the mutual position of the sources and the photometer, the experimenter ensured that the illuminance from both sources was the same, and then determined the luminous intensity of the measured source by calculation from the measured angles and distances. Several types of photometers are known.

Rumford's photometer consists of an opaque rod and a white screen. We try to place the light sources so that both shadows cast by the rod are equally dark, and the angles between the screen, rod, and sources are also the same, just on the opposite side relative to the perpendicular.

Ritchie's photometer (shown in Figure 2) compares the illuminance of two surfaces (f , g), which are illuminated from opposite sides by the standard and measured source. During measurement, we adjust the distance of the sources from the photometer.

Bunsen's photometer consists of a paper screen with a greasy/oily/waxy spot placed perpendicularly on the line connecting the light sources. The distance of the sources from the screen is adjusted until the spot and the screen are equally bright. Since the spot disperses and transmits part of the light, the spot is brighter than the rest of the screen if the illuminance of the rear side is higher than the front, and vice versa.

An alternative to Bunsen's photometer is Bothe's tangent photometer. The photometer is not located on the line connecting the lamps, but it is placed in such a way that it forms a right angle with the lamps. The position of the lamps does not change; during measurement, the plane of the photometer screen is rotated. Thus, the illuminance is balanced not by the distances of the lamps, but by the angles of incidence of light on the screen.

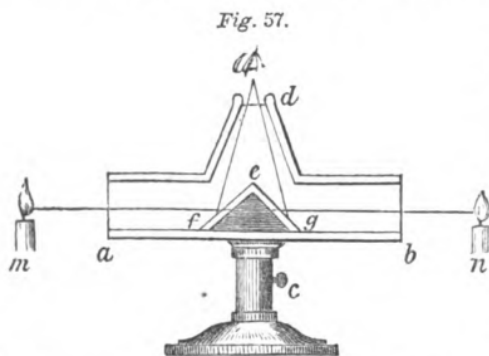


Figure 2: Ritchie's photometer²

Zollner's photometer, used in astronomy, was based on a different principle. Here, the light from a kerosene lamp was transmitted through an optical system containing a pair of Nicol prisms to the telescope's objective. The observer then compared the brightness of the stars and the lamp's image by rotating the Nicol prisms to adjust the light intensity. A Nicol prism is

²<https://en.wikipedia.org/wiki/Photometer>

made up of a pair of prisms of Icelandic spar (a type of calcite) – a birefringent material – so that the transmitted light is plane-polarized. If such light hits the second Nicol prism rotated by an angle α relative to the first one, according to Malus’s law,

$$I = I_0 \cos^2 \alpha .$$

By using more complex arrangements of birefringent materials and the dispersion of light in them, it is possible to change the color of the transmitted light, allowing us to compare lights of the same color. Comparing the illuminance from sources of different colors is considerably problematic.

Luminous intensity is a direction-dependent quantity. By measuring in multiple directions, we can determine its directional dependence. For this purpose, a goniophotometer is used, allowing measurements from every direction. This measurement holds particular significance, especially for directional light sources like LEDs or automotive headlights. By measuring luminous intensity in every direction in space, we can determine the total luminous flux emitted by the sources. A more practical method is to use an integrating sphere – a hollow sphere covered with a diffuse reflective white coating, which results in Lambertian scattering of incident light. We measure the illuminance at an output slit of the sphere, which “mixes” and “smears” the light emitted by the source with a multitude of diffuse reflections from its walls.

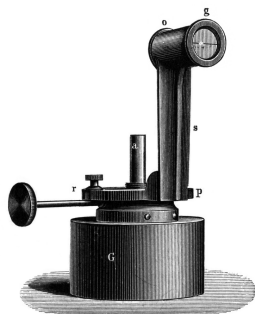


Figure 3: Hefner lamp³



Figure 4: Integrating sphere⁴

Light in every household

With the spread of gas lamps used for street lighting and factory lighting during the industrialization period at the beginning of the 19th century, a need to measure luminous intensity outside the purely scientific sphere arose. Since it was a paid service, providers were obliged to supply lamps of a given luminous intensity, and customers in large cities could usually choose from several companies. Lamps burning mainly water gas (a mixture of carbon monoxide and hydrogen formed by the reaction of water vapor and coal at high temperatures) were the main source of light until the arrival of the electric arc lamp at the end of the century

³<https://commons.wikimedia.org/wiki/File:Hefnerlampe.png>

⁴https://en.wikipedia.org/wiki/Integrating_sphere

and later the incandescent light bulb. The standard based on the original standard candles – relatively weak sources of light formed by burning fats – thus ceased to be sufficient.

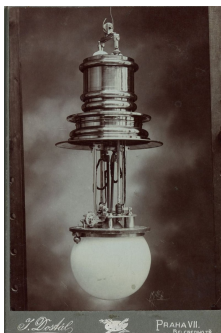


Figure 5: The arc lamp of Ludvík Očenášek⁵

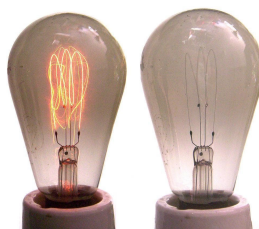


Figure 6: A carbon filament bulb, on which blackening of the bulb is observable.⁶

Efforts to enhance the definition of the standard candle and establish an international standard involved leveraging the properties of blackbody radiation. As far back as 1881, Jules Violle suggested creating a definition based on platinum at its freezing point. However, precise realization was problematic, as impurities on the surface and additives change both the freezing point and the emissivity of the surface of such a standard, reducing the precision of the implementation to tens of percent. The solution to this problem was to use another body submerged in platinum as the source. In practice, a cavity made of thorium oxide, serving as an approximation of a blackbody source, proved to be the most effective.⁷ The final proposal from 1937 was accepted as the “new candle” definition in 1946; two years later, this unit was renamed the candela. The refined definition from 1967 reads as follows:

The candela is the luminous intensity perpendicular to a surface of $1/600,000$ square meters of a black body at the temperature of solidification of platinum under a pressure of 101,325 pascals.

However, with the aforementioned approach, this definition has a precision of $3 : 10^3$, which falls significantly short of the accuracy required by the end of the 20th century. This discrepancy becomes evident particularly when compared to the definitions of other units.

Photometry Today

With the increasing accuracy of measuring specific radiometric quantities – i.e., quantities of electromagnetic radiation energy at certain wavelengths – it became possible to abandon the measurement of photometric quantities by classical visual methods and the platinum standard. This transition enables a considerable enhancement of measurement accuracy by several orders of magnitude, independent of the subjective visual perception of the experimenter. To

⁵https://commons.wikimedia.org/wiki/File:Ocenaskova_Obloukova_Lampa.jpg

⁶<https://commons.wikimedia.org/wiki/File:Carbonfilament.jpg>

⁷For a detailed description of the nearly century-long development of the methodology leading to the new definition, you may read <https://technology.matthey.com/content/journals/10.1595/003214086X3028495>.

convert radiometric quantities into photometric ones (as mathematically described below), it became imperative to ascertain the human eye's sensitivity to various wavelengths of light. The first steps in this direction were taken in the 1920s with the publication of the International Commission on Illumination (CIE) 1924 photopic function $V(\lambda)$, representing the sensitivity of daylight vision to light of different wavelengths. This field of biophysics continues to evolve, exemplified by the ongoing efforts of researchers like Stockman and Sharpe. They refine $V(\lambda)$ and elucidate the impact of color adaptation, along with measuring the color sensitivity of the eye across various conditions. Consequently, there are color sensitivity functions tailored for nighttime (scotopic) vision and for individuals with specific forms of color blindness.

The International Bureau of Weights and Measures, and thus the SI system, directly defines only the scaling constant of the conversion relationship and not its detailed behavior. It uses a monochromatic light source:

The candela, symbol cd, is the SI unit of luminous intensity. It is defined by fixing the numerical value K_{cd} of the luminous efficacy of monochromatic radiation with a frequency of $540 \cdot 10^{12}$ Hz to be 683 expressed in the unit of $\text{cd}\cdot\text{sr}\cdot\text{kg}^{-1}\cdot\text{m}^{-2}\cdot\text{s}^3$, where kilogram, meter, and second are defined using h , c , and $\Delta\nu_{\text{cs}}$.

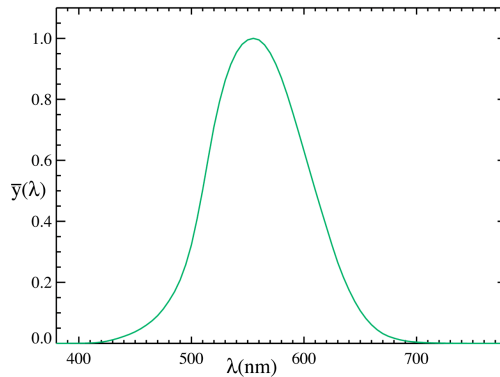


Figure 7: The curve of luminous efficiency⁸

Today's photometers usually measure illuminance (they are light meters) using a semiconductor detector of electromagnetic radiation. However, this detector measures the radiometric flux weighted by its spectral sensitivity. To convert the measured value into illuminance, it is necessary to insert a (color) filter in front of the detector so that the resulting sensitivity, combined with the detector's sensitivity, is the same as the sensitivity of the human eye (luminous efficiency). The measured value can then be converted into illuminance by multiplying it by a constant determined from calibration measurements. However, photometry is not widely used today because of advances in radiometric measuring instruments in physics. The interaction of light with matter is no longer measured using the eye. It can therefore be said that the balance between classical photometry and other physical disciplines has shifted. In several branches of physics, we still encounter the concept of photometry as measuring the properties

⁸<https://cs.wikipedia.org/wiki/Kandela>

of light. However, it is not photometry given by the human eye but by another detector, and usually, these measured values can be converted into values of radiometric quantities.

Photometry is closely related to colorimetry, which studies human color perception similarly, but with sensitivity curves for individual types of cones sensitive to red, green, and blue, and subsequent conversion to one of the color systems of the International Commission on Illumination. Today, photometry and colorimetry are important in architecture, which includes a need to meet lighting standards, as well as in image reproduction in film and graphics, and are encountered in color calibration of monitors and printers.

Derived Units

Since there is usually not enough time for photometry in high schools before graduation, let us introduce a more detailed system of photometric quantities and associated units. The main unit is luminous intensity (usually denoted by I) measured in candelas (cd), indicating the intensity of luminous flux in a particular direction per unit solid angle. The luminous flux Φ measured in lumens (lm) expresses the total amount of emitted/transmitted light energy per unit time (considering the sensitivity of the human eye). Using the definition of candela and the spectral luminous efficiency $V(\lambda)$, we can convert between monochromatic luminous flux Φ_s and radiant flux Φ_z using the relationship

$$\Phi_s(\lambda) = K_m V(\lambda) \Phi_z(\lambda),$$

where $K_m = 683 \text{ lm} \cdot \text{W}^{-1}$. For light with a continuous spectrum, it is necessary to integrate this relationship – ultimately resulting in a weighted average of the spectral radiant flux weighted by the eye's sensitivity and multiplied by the defining constant. For the luminous intensity of the source, we then have the relationship

$$I = \frac{d\Phi}{d\Omega},$$

where $d\Phi$ is the luminous flux directed into a solid angle of size $d\Omega$. Candela thus corresponds to luminous intensity ($\text{W} \cdot \text{sr}^{-1}$) in radiometric quantities. Luminous flux is also derived from luminous energy Q_V

$$\Phi = \frac{dQ_V}{dt}$$

in the unit of $\text{lm} \cdot \text{s}$, similar to the energy of radiation in the unit of joules.

While luminous flux describes a source of radiation, illuminance E is given as the luminous flux incident on a unit area

$$E = \frac{d\Phi}{dS}.$$

Therefore, it decreases inversely with the square of the distance from the source and is directly proportional to the cosine of the angle between the direction of incident radiation and the normal to the surface where the illuminance is being measured. The unit of illuminance is lux $1 \text{ lx} = 1 \text{ lm} \cdot \text{m}^{-2}$. A similar quantity is luminous exitance M , describing the luminous flux emitted by a light source per unit of its surface area; here, instead of lux, the unit $\text{lm} \cdot \text{m}^{-2}$ is usually used. In photography, we also encounter the luminous exposure H_V given by the amount of light that falls on the photographic film over the time of exposure t

$$H_V = \int E dt.$$

Table 1: Prefixes of the SI system

quetta	ronna	yotta	zetta	exa	peta	tera	giga	mega	kilo	hecto	deca
Q	R	Y	Z	E	P	T	G	M	k	h	da
10^{30}	10^{27}	10^{24}	10^{21}	10^{18}	10^{15}	10^{12}	10^9	10^6	10^3	10^2	10^1
quecto	ronto	yocto	zepto	atto	femto	pico	nano	micro	milli	centi	deci
q	r	y	z	a	f	p	n	μ	m	c	d
10^{-30}	10^{-27}	10^{-24}	10^{-21}	10^{-18}	10^{-15}	10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^{-1}

A related quantity is luminance L , which describes the amount of light emitted per unit area of a light source into a given solid angle (in a particular direction)

$$L = \frac{dI}{dS \cos \alpha},$$

where α is the angle between the normal to the surface and the direction of radiation. Luminance has a unit of $\text{cd}\cdot\text{m}^{-2}$ and is a quantitative description of what we would call the brightness of a light source in everyday life.

What else does SI bring us?

In addition to the definitions of the seven base units using seven constants $\Delta\nu_{\text{Cs}}$, c , h , N_{A} , k_{B} , e , K_{cd} , the SI system also introduces units for measuring angles. The fundamental units of the plane and solid angles are the radian $1 \text{ rad} = 1 \text{ m}/1 \text{ m}$, defined as the ratio of the length of a circular arc which subtends this angle to the radius, and the steradian $1 \text{ sr} = 1 \text{ m}^2/1 \text{ m}^2$, defined as the ratio of the area of a spherical cap to the square of its radius. These units possess a dimension of 1 in the SI system, rendering them dimensionless. However, in certain contexts, they are specified to avoid ambiguity, such as in the unit of angular velocity $\text{rad}\cdot\text{s}^{-1}$, or in the case of the previously mentioned lumen $\text{cd}\cdot\text{sr}$. Nonetheless, angular measures are often expressed in degrees or square degrees. To convert, we use the definition of the degree $360^\circ = 2\pi \text{ rad}$ using the full circle. Unlike SI units, the degree is not divided in powers of 10, but into arc minutes with $1^\circ = 60'$ and arc seconds with $1' = 60''$.

SI further introduces its own system of prefixes for multiples and submultiples of the base units, based on powers of ten. These prefixes, as listed in Table 1, are attached to the names and symbols of units – for example, $1 \text{ nm} = 1 \cdot 10^{-9} \text{ m}$ is a nanometer. For historical reasons, the kilogram unit itself is defined as a thousand times the gram, and its multiples use prefixes derived from gram, such as $10^{-6} \text{ kg} = 1 \text{ mg}$ instead of $1 \mu\text{kg}$.

Lastly, SI specifies the usage of units and their typography. Unit names begin with lowercase letters, prefixes are appended without hyphens, forming a single word. Each value of a physical quantity is expressed as the product of its numerical value and the unit it is expressed in, like $m = 1 \text{ kg}$. The notation of quantities employs italics, while units use the regular font. A small space separates the numerical value and the unit, with the numerical value listed first. When expressing the value of a quantity and its uncertainty, parentheses must enclose them, such as $m = (1.00 \pm 0.02) \text{ kg}$. Another option is the notation $m = 1.00(2) \text{ kg}$, where the value in parentheses indicates the error in the last digits of the stated value. Similarly, in tables and graphs where only numerical values are plotted, the units used must be specified, usually in the form of a ratio m/kg . The rules for formatting numerical values also apply, for example, $-0.123\,456\,789\,0$, where in Czech and Slovak, a decimal comma is used, and a space is used as a thousands separator. For multiplication and division of units, any of ab , $a\,b$, $a \times b$,

$a \cdot b$, a/b , $\frac{a}{b}$, ab^{-1} , etc., are acceptable. In the case of dimensionless physical quantities, only the numerical value is used without a unit, so it is not possible to use a prefix, and the order must be expressed numerically by multiplying by the corresponding power of ten. In some cases, however, it is appropriate to use the respective unit, such as in the case of the Earth's rotational slowdown $23 \mu\text{s}/\text{yr}$.

Other unit systems

In addition to classical unit systems, we can encounter other systems in physics, usually specific to a given application. In astrophysics, we typically encounter units based on the Earth's orbit around the Sun – the astronomical unit⁹ (and the derived unit parsec¹⁰), the mass of the Sun $1 M_{\odot}$, and the day equal to $1 \text{ d} = 86\,400 \text{ s}$, where the SI second is used. These units have practical advantages from several points of view. It is significantly easier to compare stars and planets to the Sun and the Earth without constantly converting numbers with large orders of magnitude. Usually, only the relative comparison of measured quantities holds importance. Moreover, this system historically prevailed when the measurement of distance from the Earth to the Sun lacked precision. Even today, the mass of the Sun serves as a more accurate unit of mass in space compared to the kilogram. Measuring mass is done by the effect of gravitational force on surrounding bodies, and the value of the gravitational constant $G = 6.674\,30(15) \cdot 10^{-11} \text{ N}\cdot\text{m}^2\cdot\text{kg}^{-2}$ is known with less precision than the value of the so-called standard gravitational parameter $GM_{\odot} = 1.327\,124\,400\,42(10) \cdot 10^{20} \text{ m}^3\cdot\text{s}^{-2}$. One of the further advantages of using such units is the simplification of calculations using Kepler's third law – by using the mass of the Sun, the astronomical unit, and the year as units, we can utilize the relationship in the form

$$\frac{a^3}{P^2} = M + m.$$

In special relativity, we encounter so-called geometrized units. By setting the speed of light and the gravitational constant to be equal to one, equations are simplified – these quantities will not appear in them, and interesting phenomena will occur at values of quantities in the order of unity. Additionally, by introducing unit values of the Boltzmann constant, reduced Planck constant, and vacuum permittivity, we obtain one of the possible realizations of the so-called Planck units. Similar caution as with the introduction of electrical units must be exercised here, considering the potential for alternative definitions involving various $n\pi$ multiples of these quantities. Planck units find extensive application in theoretical physics, particularly in theories attempting to simultaneously describe all fundamental force interactions – so-called theories of everything. Unit values of quantities are tied to exploration of various “barriers” of physics, whether quantum or relativistic in nature.

In atomic and molecular physics, the so-called Hartree units are used by introducing unit values of the reduced Planck constant \hbar , charge and mass of the electron e , m_e , and $4\pi\epsilon_0$ – a multiple of the vacuum permittivity. In these units, the radius of the ground state of the hydrogen atom in the Bohr model has a value of one and serves as the unit of length

$$1 a_{\text{B}} = 4\pi\epsilon_0 \frac{\hbar^2}{m_e e^2}.$$

⁹Originally defined as the semi-major axis of the orbit of the Earth-Moon barycenter around the Sun, which has been refined several times. In 2012, it was finally tied to the SI as $1 \text{ au} = 149\,597\,870\,700 \text{ m}$.

¹⁰Parsec is used for stellar distances, defined by the parallax method of measurement as the distance at which 1 au subtends an angle of one arcsecond, i.e., $1 \text{ pc} = 648\,000/\pi \text{ au}$

Similarly, twice the energy of the ground state of the hydrogen atom serves as a unit, named the hartree

$$1 E_h = \frac{\hbar^2}{m_e a_B^2}.$$

You might have encountered these units in last year's series dedicated to computational physical chemistry.

In particle physics, we encounter units based on the energy unit electronvolt – the energy gained by an electron accelerated by one volt $1 \text{ eV} = 1.602\,176\,634 \cdot 10^{-19} \text{ J}$, and the speed of light¹¹. For example, the mass of the Higgs boson is given as $125.11 \pm 0.11 \text{ GeV} \cdot \text{c}^{-2}$. In nuclear physics, when describing strong nuclear interaction, it is advantageous to use units determined by fixing the values of the proton mass, speed of light, and reduced Planck constant. Generally, to describe quantities derived from n basic units, we need n appropriate defining constants chosen according to the needs of the specific field.

Other scientific disciplines adopt their own units wherever it becomes necessary to quantitatively express a property. In computer science, the bit or its multiple the byte $1 \text{ B} = 8 \text{ b}$ serves as a unit of information quantity. In pharmacology, international units (I.U.) are used to describe the biological activity of substances. We encounter them when describing the amount of vitamins, hormones, drugs, etc., instead of their mass, as the effect on humans described by these units depends on the form and method of administration. Additionally, chemically different substances or mixtures with different molecular masses can have qualitatively the same pharmacological effect. The measure of 1 U.I. is defined for individual substances by the Expert Committee on Biological Standardization of the World Health Organization. Similar units are also encountered in everyday life – Scoville units for measuring spiciness or a standard drink for alcohol dose. Lastly, there is a range of units used in economics and other social sciences, derived from human activity, such as the man-hour for a measure of performed work, man-night in hotel management, micromort in insurance, or equivalent inhabitants in wastewater treatment.¹²

Conclusion

In conclusion, I would like to thank all the participants who stuck with me until the end despite my occasional tardiness. I believe you had the opportunity to refresh your knowledge in various branches of physics and often learn something new as well. For myself, much of the presented information was new, and especially the descriptions of the function of individual instruments sometimes took me a few days to absorb before I could understand and present them in (hopefully) understandable form. Finally, I would like to thank all the technicians and engineers, without whose detailed knowledge and persistent work, physics research as we know it would not have been possible.

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¹¹Optionally also Boltzmann and reduced Planck constants.

¹²A humorous “peak” of the human need to quantify things is the penrig unit – an acronym for PENis RIGidity describing the degree of erection.