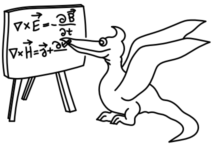

Serial: Electric current

In this penultimate chapter of the series, we will look at the last major area of measurement that the average mortal encounters – the measurement of electromagnetic quantities. This field has a rich but also relatively modern history, resulting in more than one known system of units and also many quantities with units named after famous physicists who contributed to the advances of electricity and magnetism.

Ampere

Historical development of electromagnetism

Electricity and magnetism have fascinated humans for millennia. As far back as the 6th century BC, Thales of Miletus noted the curious interaction between charged amber and feathers, as well as the magnetic pull between stones containing magnetite and iron. Despite early observations, practical applications remained elusive until the 11th century when Chinese sailors devised the compass, a breakthrough that revolutionized navigation. By the 12th century, European scholars gained access to this navigational tool through Arab seafarers, sparking a period of intensive study and refinement. Over the next few centuries, pioneers like William Gilbert made significant strides, distinguishing between magnetism and static electricity and mapping out the Earth's magnetic field. In the early 18th century, Charles Francois du Fay built upon these foundations, proposing the "two-fluid" theory of electricity.

Advances in this period are also closely bound to the invention of the electrostatic generator separating positive and negative charge and the Leyden jars allowing this charge to be collected, transferred, and stored, often to entertain audiences during public electrical shows, usually by discharging the charge through a line of people holding hands. The first measuring "apparatus" were thus the experimenters themselves, who, for example, sorted substances into conductors and non-conductors according to the perceived strength of the electric discharge. The first serious experiment was carried out in 1748 by W. Watson and W. Cavendish and aimed to determine the speed of the propagation of electricity. Their conclusion, with respect to the measurement methods, was that electricity traversed a several-kilometer-long circuit instantaneously. The first real measuring instrument was the electroscope constructed in 1748 by the Frenchman Jean-Antoine Nollet. His experiments contributed to the proposition of J. Priestley in 1767, suggesting that the force between charged bodies is inversely proportional to the square of their distance. This relation is commonly known as Charles-Augustin Coulomb's Law, established experimentally in 1785 using a torsion balance. In the following period, Siméon-Denis Poisson and Pierre-Simon Laplace developed equations describing the electric potential for known charge distributions. Today, the electrometer (successor to the electroscope) is an apparatus for determining the magnitude of charge by the mutual repulsion of two metallic hands enclosed in a vacuum.

Italian physicist Alessandro Volta made another significant contribution to the research of dynamical phenomena. He built upon the experiments of J. Sulzer and L. Galvani, who had

[1] `https://en.wikipedia.org/wiki/Electrometer`

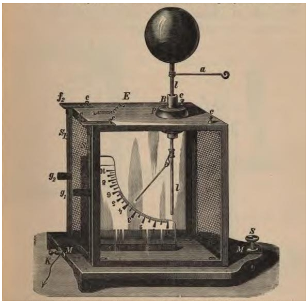

Figure 1: Kolbe's electrometer – the hand of the device is deflected by electrostatic interaction[1]

studied the reaction of tissues when in contact with two different metals. By substituting animal tissue with paper soaked in saltwater between silver and zinc plates and then stacking many such units on top of each other, he demonstrated the voltaic pile, which served as the precursor to today's batteries. Shortly after this discovery of a source of constant electrical voltage, W. Nicholson and A. Carlisle demonstrated the decomposition of water into oxygen and hydrogen under the influence of electricity. Volta's pupil, L. Valentino, is the first to use electricity for electroplating. British chemist Humphry Davy used electricity to isolate new chemical elements – sodium and potassium in 1807 and five other elements the following year, laying the foundations for electrolysis. His pupil, Michael Faraday, suggested this term for chemical processes induced by electricity, and additionally, in 1833, Faraday formulated the law of electrolysis

$$M = \frac{QM_\mathrm{m}}{F\nu}\,,$$

where M is the mass of a substance of molar mass M_m formed at the electrode after a charge Q has been passed through the electrolyte, transferring ν moles of electrons per mole of substance.[2]

Another major breakthrough (also in the context of measurement) was Hans Oersted's discovery in 1820. During his lecture, he noticed how in the presence of a current-carrying conductor, a compass needle was deflected perpendicularly to this conductor, thus connecting electricity and magnetism. The publication of this observation triggered the keen interest of physicists, and after only a week, André-Marie Ampère observed the interaction between two parallel current-carrying conductors. Shortly afterward, Arago observed how iron filings arranged around the conductor in concentric circles. That very same year, Jean-Baptiste Biot and Félix Savart established a relation for determining the magnetic field of a system of conductors, and two years later, Ampère published a more general and rigorous version – now known as Ampère's law. That led Poisson and Green to introduce the magnetic vector potential. The discovery of the generation of a magnetic field around conductors carrying an electric current leads in short succession to inventions such as the electromagnet, the telegraph, and the electric motor. However, the crucial outcome for us is the ability to quantitatively measure electric current resulting from the effect of a generated magnetic field.

[2] Usually there is a change of the oxidation number of the atom by ν.
[3] `https://commons.wikimedia.org/wiki/File:Pila_di_Volta.jpg`

Figure 2: Voltaic pile – source of constant DC voltage[3]

Another breakthrough came as early as 1831 when Micheal Faraday observed how a variable current in one circuit generated a current in a different circuit, isolated from the first one. After a thorough investigation, he discovered that a variable magnetic field induces an electric voltage in a closed circuit. In 1834, he published the now well-known Faraday's law of electromagnetic induction. As early as 1846, W. Weber attempted to create a unified theory of electromagnetism, but it was not until 1864 that James Clark Maxwell succeeded. Maxwell achieved this after a decade of theoretical work, formulating Maxwell's equations, although in a mathematical form that is almost unrecognizable today. During the same time, Weber, Kohlrausch, and Kirchhoff proposed the hypothesis that electricity propagates at the speed of light. In 1867, Lorentz proposed the theory that light itself is of electromagnetic nature. That was proved experimentally in 1887 by Henri Hertz, who created radio waves and demonstrated that they possess the same properties as light.

These groundbreaking discoveries in fundamental physics naturally paved the way for practical inventions. For instance, Guglielmo Marconi developed the radio in 1894, while the French instrument maker Hippolyte Pixii constructed the first dynamo in 1832.

In 1867, the first dynamo able to generate electricity for industrial use was built independently by Ch. Wheatstone, W. von Siemens, and S. A. Varley.[4]

Finally, let us add the definition of the unit of electric current. As we will describe at the end of this chapter, the situation is complicated due to the different approaches to the choice of the system itself. The original definition of the unit of electric current in the so-called electromagnetic system in CGS units was the biot, also called the abampere

One biot is the electric current that causes two parallel conductors distanced 1 cm to

[4]For those interested in a detailed historical account of the development of ideas not only about electricity and magnetism and the inventions, debates, and rivalries associated with them, I recommend the YouTube channel *Kathy Loves Physics & History*

[5]`https://commons.wikimedia.org/wiki/File:Wechselstromerzeuger_Crop_LevelAdj.jpg`

[6]`https://commons.wikimedia.org/wiki/File:Jedlik_motor.jpg`

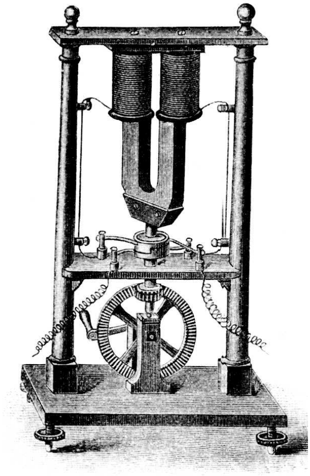

Figure 3: Hippolyte Pixii's dynamo – the U-shaped magnet is turned by a crank under a pair of coils[5]

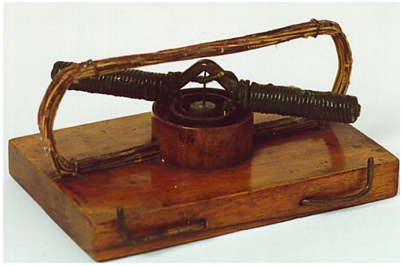

Figure 4: Jedlik's electric motor[6]

> exert on each other a force of 2 dyn[7] per centimeter of the length of the conductors.

However, at the end of the 19th century, it was impractical to measure such small forces, thus the so-called international ampere was introduced, defined as follows

> One ampere is the electric current which, when flowing through an aqueous solution of silver nitrate, deposits silver at a rate of $1.118\,\mathrm{mg{\cdot}s^{-1}}$.

Furthermore, the practical realization of electrical resistance was introduced using the resistivity of a mercury column, and the electrical voltage by fixing the voltage of the Clark, and later, the Weston cell. By increasing the accuracy of the measurements of the forces between coils using the Ampere balance, it was possible to come back in 1946 to almost the original definition

> The ampere, a unit of electric current, is the constant current that is maintained in two infinite parallel conductors of a negligible circular cross-section at a mutual distance of 1 m, and which, in a vacuum, generates between them a force of $2\cdot10^{-7}\,\mathrm{N}$ per meter of length of the conductors.

Analog measuring instruments

Oersted's discovery of the deflection of a magnet near a current-carrying conductor laid the foundation for measuring electromagnetic quantities. The simplest measuring instrument to emerge was the tangent galvanometer. It consists of several loops of copper wire wound on a round non-magnetic, and non-conducting frame oriented vertically. Placed horizontally in the middle of this coil is a compass needle. Before measurement, we must rotate the apparatus

[7] $1\,\mathrm{dyn} = 10^{-5}\,\mathrm{N}$

so that the compass needle lies in the plane of the coil. Then, we apply the current we want to measure to the coil and measure the deflection of the needle. This instrument compares the magnetic field in the axis of the coil with Earth's magnetic field.

More advanced instruments utilize the field of a permanent magnet instead of relying on Earth's magnetic field. An example is the D'Arsonval galvanometer, which features two permanent magnets between which a square coil is fixed on an axis. Electric current is supplied to this coil via torsion springs, which also counteract the coil's deflection caused by the current. A pointer attached to the coil's frame allows for reading the current value from a scale. To enhance sensitivity, an iron cylinder is positioned at the coil's center, amplifying the coil's magnetic field by up to three orders of magnitude.

An alternative inverse design involves a moving magnet positioned within a static coil. Another permanent magnet's field serves as a balancing force in this configuration. Essentially, it operates similarly to a tangent galvanometer. Compared to D'Arsonval's design, it can handle larger currents since the coil can be wound with thicker wire, eliminating the risk of melting.

In general, instruments designed for measuring electric current are called ammeters. These devices typically have a small internal resistance (the resistance of the coil itself) and are connected in series within the circuit, allowing the entire measured current to flow through them. If we were to connect a resistor with a large resistance R in parallel within the circuit and measure the current passing through it, we would effectively create a voltmeter. The voltage measured would abide by Ohm's law $U = RI$. With more intricate circuitry, it is feasible to transform a galvanometer into an ohmmeter. However, all the instruments mentioned thus far are primarily capable of measuring direct current (DC). Standard alternating current (AC) would induce changes too rapid for these instruments to (mechanically) respond to, resulting in the display of only average readings.

For measuring alternating quantities, we use ammeters containing two separate iron parts. A fixed coil induces a magnetic field in its iron core, and a second piece of iron suspended above the core on a spring is attracted to it, thereby deflecting a needle. The force between the irons is proportional to the square of the current in the coil, so if the spring has an intrinsic period of oscillation significantly greater than the period of the applied alternating current, the ammeter indicates an average value of I^2, and the scale plots the root mean square value of the current in non-linear intervals.

A ballistic galvanometer is also associated with a long response time. Its slow response is due to the large moment of inertia of the mechanical parts. We can use it to measure integral quantities, such as the quantity of overflown charge when discharging a capacitor.

Finally, let us mention one more significant measurement method, especially for measuring parameters of various components – bridges. We can use these to determine the parameters of an unknown electrical component from the values of other components, at least one of which must have an adjustable value. The most basic is the Wheatstone bridge composed of three known resistors and one unknown resistor, whose electrical resistance we want to measure.

The galvanometer in the middle of the bridge will show zero current passing through if the resistors are balanced and the following will be true

$$R_x = \frac{R_3 R_2}{R_1}\,.$$

[8] `https://commons.wikimedia.org/wiki/File:Tangent_galvanometer_Philip-Harris_top1.jpg`

[9] `https://commons.wikimedia.org/wiki/File:A_moving_coil_galvanometer._Wellcome_M0016397.jpg`

[10] `https://commons.wikimedia.org/wiki/File:Wheatstonebridge.svg`

Figure 5: Tangent galvanometer[8]

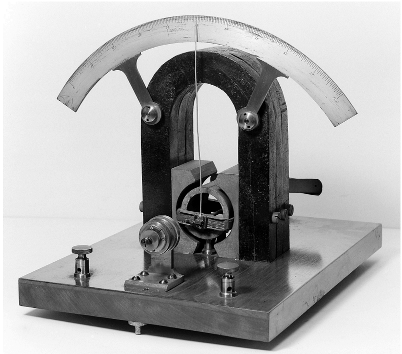

Figure 6: D'Arsonval galvanometer[9]

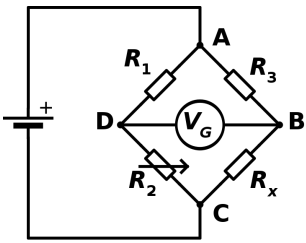

Figure 7: Diagram of the Wheatstone Bridge[10]

To achieve this condition, we measure the resistance of resistor R_2.

Similarly, we can use the Maxwell bridge to determine the inductance L and the Wien bridge to measure the electrical capacitance C. Both bridges utilize an AC voltage source for measurement and can pinpoint the internal resistance of real components in addition to L and C.

Digital measurements

Digital measuring instruments operate by converting analog voltage values into digital values sampled over time. An analog-to-digital (AD) converter, which plays a crucial role in this process, is characterized by two important parameters: the number of bits n and the sampling frequency f. The device can distinguish 2^n voltage values between U_{min} and U_{max}. We often choose $U_{\mathrm{min}} = 0$ or $U_{\mathrm{min}} = -U_{\mathrm{max}}$. The conversion thus produces a rounding error of half the resolution of the instrument

$$\Delta U = \frac{U_{\mathrm{max}} - U_{\mathrm{min}}}{2^n} \,.$$

An essential design requirement is the linearity of the device so that each bit corresponds to the same voltage increment. Verification and potential calibration are carried out using a more precise instrument. If we are measuring the time dependence of the voltage, we will certainly be interested in the value of the sampling frequency. For a good description of the actual waveform, we require that the sampling frequency be at least twice as high as the highest frequency of the signal being measured (i.e., the time between measurements is at most half the characteristic time of changes in the value of the signal) – this is called the Nyquist theorem. Otherwise, the actual frequencies may be misinterpreted – if a sinusoidal signal with frequency f_0 is sampled with frequency $f < f_0$ the result of the measurement will look like a sinusoid with frequency $f_1 = f_0 - f$ – the so-called aliasing. With an AD converter, we can measure both DC and AC quantities. In the latter case, however, we will get the voltage dependence on time, not directly the amplitude of the AC voltage. Therefore, in many cases, the AC voltage is initially rectified and smoothed within the instrument to obtain a DC voltage, which is then measured.

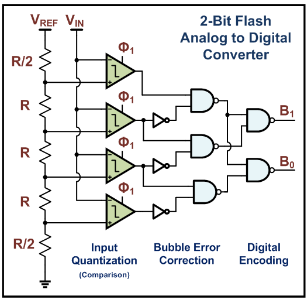

Figure 8: Diagram of flash ADC converter[11]

[11] https://commons.wikimedia.org/wiki/File:Flash_ADC.png

The easiest to explain is the flash AD converter, comprised of a sequence of identical resistors dividing the reference voltage as in the figure 8. These fractions

$$U_k = U_{\mathrm{ref}} \frac{k + \frac{1}{2}}{2^n}$$

are then compared with the measured voltage U_{in} by transistors after a possible amplification of the difference of these voltages by a preamplifier. A series of coupled transistors thus expresses the measured voltage in unary notation, which is converted to binary using logic gates. This configuration is capable of measuring at high frequency, but a large number of resistors and transistors are needed to obtain higher resolutions – the n-bit converter contains 2^n resistors and transistors. A commonly used reference voltage source is a Zener diode connected in the forward bias direction. That is because once the critical breakdown voltage is surpassed, its volt-ampere characteristic becomes extremely steep, which makes it possible to create a circuit stabilized just to this value with stability up to $1 : 10^6$. A second prevalent option involves directly utilizing the voltage drop across a semiconductor junction.

Structurally different is the successive-approximation AD converter. This one uses a digital-analog converter for digitizing, which produces a voltage value and then compares it with the measured value. The main advantage is having fewer components – for an n-bit converter, a set of n sources with voltages $U_0 \cdot 2^k$ for k from 1 to n, the corresponding n switching transistors and a single comparison transistor are sufficient. That can be crucial, especially for higher resolutions. For instance, a 12-bit flash converter would necessitate approximately $2^{12} \doteq 4\,000$ transistors, while a 16-bit converter could require up to $2^{16} \doteq 66\,000$. During measurement, the comparison converter, employing a sample circuit and a hold circuit, initially "copies" the voltage value at a specific moment and then compares it to the values from the digital-to-analog (DA) converter through binary scanning. Consequently, the measurement process is considerably slower compared to a flash converter.

Other alternatives are integrating converters, which use the capacitor charge/discharge time to measure the circuit voltage and determine the voltage value by computation. These devices have the worst time resolution but high resolution and accuracy. Therefore, they are unsuitable for tasks such as sound recording but are ideal for use in classical multimeters.

In addition to directly measuring voltage, modern digital voltmeters can measure many other quantities by converting them into voltage first. The simplest method is to measure current by assessing the voltage across a known resistor. Likewise, electrical resistance, capacitance, and inductance can be measured using bridges in a similar way to analog instruments. It is possible to measure changes in parameters with high accuracy by determining the change in voltage across the bridge. That allows us to measure other quantities, such as temperature using the Seebeck effect on a thermistor, pressure using the piezoelectric effect, or the magnetic field using the Hall effect. We can encounter AD converters outside the usual measuring devices. For instance, they are used to convert the excited charge resulting from the photoelectric phenomenon on a camera chip into pixel values on a photograph.

Precision comes with quantum physics

Another significant advance in measurements and their standardization occurred in the second half of the 20th century with the advancement of low-temperature physics and the investigation of superconductivity and related quantum phenomena. One of the most significant phenomena for metrology is the quantum Hall effect.

First, let us explain the normal Hall effect, which was discovered by E. Hall in 1879. Assume we have a block of semiconducting material as in figure 9 of width W, length L, and thickness t. If we place this block in an external magnetic field with magnitude B_z oriented perpendicular to the block and attach electrodes to its edges through which we let a current I_x flow, this field tends to deflect the charge carriers in the direction of the y axis. This results in a charge distribution in the block between the left and right walls whose electric field E in the direction of the y axis compensates for this deflection. From the relation for a force acting on a charged particle, we have

$$0 = \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad \Rightarrow \quad E_y = vB_z \,,$$

where $\mathbf{v} = (v, 0, 0)$ is the velocity of motion of the particles. This velocity is related to the current via the particle density of charge carriers n, their charge, usually of the size of the elementary charge e, and the cross-section of the conductor $S = tW$ as

$$I_x = nevS = nevtW \,.$$

By substituting, we get the relation

$$I_x = \frac{netdE_y}{B_z} = \frac{net}{B_z} U_y \quad \Rightarrow \quad R_\mathrm{H} = \frac{U_y}{I_x} = \frac{B_z}{net} A_\mathrm{H} \,,$$

where we have also inserted the relation between the voltage and the strength of the homogeneous electric field $U = Ed$ and R_H is the so-called Hall resistance. The dimensionless quantity A_H added to the relation is termed the dissipation factor, which varies between 1 and 2 depending on the material utilized. Thus, by measuring the current and voltage in the perpendicular direction, we can determine the magnitude of the magnetic field.

In 1980, while investigating the Hall resistance in a thin film, i.e., in 2D configurations, K. von Klitzing found with measurements on MOSFET transistors that in a pure silicon semiconductor at low temperatures and under strong magnetic fields the Hall resistance does not follow the classical continuous relation but only takes on certain values

$$R_\mathrm{H} = \frac{1}{\nu} \frac{h}{e^2} \,,$$

where h is Planck's constant and ν is an integer. This relation, upon substitution, corresponds to the dependence of the particle charge density – specifically of electrons –given by

$$nt = n_\mathrm{2D} = \nu \frac{eB}{h} \,.$$

But why should this value be discrete? There are $N = Sn_\mathrm{2D}$ electrons on the surface S of our layer, and the magnetic flux $\Phi = BS$ flows through it. Moreover, quantum magnetic field theory implies that $2\Phi_0 = h/e$ is the quantum of the magnetic induction flux. Thus, after substitution, we get the relation

$$N = \nu \frac{\Phi}{2\Phi_0} \quad \rightarrow \quad \Phi_0 = \nu \frac{\Phi}{2N} \,.$$

One quantum of magnetic flux is equal to the flux associated with an integer multiple of electrons. The presence of the factor of two in the equation is attributed to the two distinct spin states that an electron can occupy in a magnetic field. The energy perspective is also intriguing, as electrons can only occupy discrete energy levels in the presence of a magnetic field

$$E_n = hf_\mathrm{L}\left(n+\frac{1}{2}\right)\,,\quad f_\mathrm{L} = \frac{qB}{2\pi m}$$

where f_L is the Larmor frequency – the frequency of the circular motion that a charged particle performs in a magnetic field. However, the highest energy electrons can acquire corresponds to the Fermi energy E_F – only some states for which $E_n < E_\mathrm{F}$ will be occupied. In practice, the individual levels are not completely sharp because of the scattering of electrons from their orbits due to the thermal motion of the lattice atoms and impurities. When the value of the magnetic flux density B is such that this extended level overlaps with the Fermi energy, the electrons of this level are able to move, and the material becomes conductive with non-zero resistance. Otherwise, the material is superconducting with zero resistance. These are the precise values of B for which ν in the relation for Hall resistance changes from one value to another.[12] For his experimental work and verification of the independence of the quantum Hall effect from many variables, von Klitzing was awarded the Nobel Prize in 1985, and the value

$$R_\mathrm{K} = \frac{h}{e^2}$$

was named the von Klitzing constant. The dependence of the longitudinal (ordinary) and transverse (Hall) resistance in a real material is shown in Figure 10. In the weak fields, we see a directly proportional dependence of the Hall resistance on the magnetic flux density and a constant value of the longitudinal resistance, which we expected from the non-quantum description.

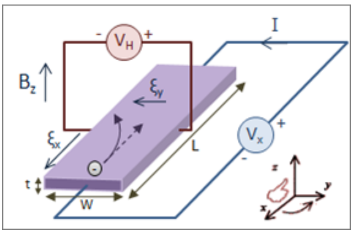

Figure 9: The geometry of the Hall effect[13]

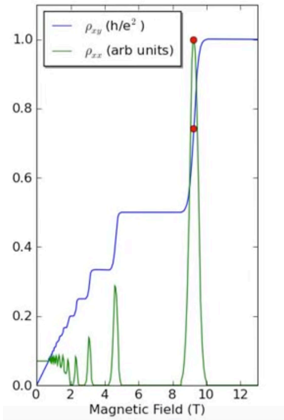

Figure 10: The dependence of resistance on magnetic flux density[14]

[12] A nice illustrative video can be found on Wikipedia `https://en.wikipedia.org/wiki/File:QuantumHallEffectExplanatioWithLandauLevels.ogv`

[13] `https://commons.wikimedia.org/wiki/File:Hall_Effect_Measurement_Setup_for_Electrons.png`

[14] `https://en.wikipedia.org/wiki/Quantum_Hall_effect`

Another significant phenomenon in standardization is the Josephson effect. In superconductors, electrons propagate in pairs known as Cooper pairs. A Josephson junction is created by sandwiching a thin layer of insulator or normal conductor between two superconductors. The Josephson phenomenon describes the quantum tunneling of the Cooper pair of electrons through this barrier and was described theoretically in 1962 and verified experimentally in 1963. In 1973, B. Josephson was awarded the Nobel Prize. If we denote the phase difference of the Cooper pair wave function at the barrier between the superconductors by δ and we maintain a voltage U across the superconductors, then the current flowing through the transition is given as

$$I = I_\mathrm{c} \sin\left(\delta(t)\right) ,$$

where I_c is the critical transition current, and the phase transition between the environments is

$$\frac{\partial \delta}{\partial t} = 2\pi \frac{2eU}{h} ,$$

where the fraction $2e/h = K_\mathrm{J}$ is called the Josephson constant and is also the inverse value of the magnetic flux quantum. If there is no voltage at the junction, the current takes on a constant value between $-I_\mathrm{c}$ and I_c. However, if a constant voltage U_0 is applied to the junction, the phase jump will vary linearly in time, resulting in a sinusoidal current waveform with an amplitude I_c and a frequency $K_\mathrm{J} U_0$. Hence, the Josephson junction converts voltage to a frequency we can measure accurately using an atomic clock. The transition can also work the other way around; by bringing in a monochromatic microwave signal, the voltage at the transition and the corresponding phase leaps are quantized

$$U = n \frac{f}{K_\mathrm{J}} .$$

This principle is behind the most accurate voltage standards and replaced Weston's electrolytic cell as the voltage standard, increasing the accuracy of voltage standards from $1 : 10^6$ to $1 : 10^9$ within a decade. Today, the NIST's programmable $0\,\mathrm{V}$ – $10\,\mathrm{V}$ standard constituted of 300 000 superconducting Josephson junctions cooled by liquid helium has an accuracy of $1 : 10^{10}$.

These phenomena enabled the introduction of the conventional electrical unit in 1990 by fixing the values of the frequency of the hyperfine transition of caesium to conform to the SI system and the values of the Josephson and von Klitzing constants at $K_\mathrm{J}^{90} = 483\,597.9\,\mathrm{GHz{\cdot}V^{-1}}$ and $R_\mathrm{K}^{90} = 25\,812.807\,\Omega$. The accuracy was set by comparing the Josephson voltage standard and the electrical resistance standard, implemented using the quantum Hall effect and atomic clocks. This procedure was formally transferred to the SI in 2019 as a practical implementation of the new definition of the ampere

> The ampere, symbol A, is the SI unit of electric current. It is defined by fixing the numerical value of the elementary charge e at $1.602\,176\,634 \cdot 10^{-19}$ expressed in units of As, where a second is defined by $\Delta\nu_{Cs}$.

The value of Planck's constant and of the electron charge used in the SI definitions give slightly different numerical values than K_J^{90} and R_K^{90}. Indeed, in certain cases, it is feasible to directly measure electric current by quantifying the number of individual electrons passing through a surface within a measured time interval.

Derived quantities

Besides the ampere, we encounter many other units in electricity and magnetism. Almost every quantity has its unit named after a physicist who has contributed to the development of the field. In electrostatics, we find the unit of electric charge, the coulomb $1\,\mathrm{C} = 1\,\mathrm{A{\cdot}s}$. As mentioned in the historical introduction, one method for measuring the charge is the electrometer, and another is the ballistic galvanometer, which integrates the current flowing through it. Usually, for electrical components, we encounter minor charges at about mC and smaller. Batteries have larger charges, but we often use composite units mA·h. Common alkaline batteries carry about $1\,200\,\mathrm{mAh} = 4.32\,\mathrm{kC}$. The difference of electrostatic potentials, i.e., the electric voltage, has the unit volt $1\,\mathrm{V} = 1\,\mathrm{J{\cdot}C^{-1}} = 1\,\mathrm{kg{\cdot}m^2{\cdot}s^{-3}{\cdot}A^{-1}}$. For a description of its measurement, see the previous chapters. The last of the electrostatic quantities with its own unit is the electrical capacitance, which describes the ratio between the voltage and charge on the plates of the capacitor $1\,\mathrm{F} = 1\,\mathrm{C{\cdot}V^{-1}} = 1\,\mathrm{kg^{-1}{\cdot}m^{-2}{\cdot}s^4{\cdot}A^2}$. We typically measure the voltage and current when the capacitor is charging/discharging or utilize the capacitance with AC current to determine the capacity. Capacitance values typically range from pF to μF. In the context of simple circuits, we also encounter the unit of electrical resistance ohm $1\,\Omega = 1\,\mathrm{V{\cdot}A^{-1}} = 1\,\mathrm{kg{\cdot}m^2{\cdot}s^{-3}{\cdot}A^{-2}}$ and electrical conductivity siemens $1\,\mathrm{S} = 1\,\Omega^{-1} = 1\,\mathrm{kg^{-1}{\cdot}m^{-2}{\cdot}s^3{\cdot}A^2}$, which are closely related to the material properties and the geometric shape of the body. Their magnitude is typically determined from their definition by simultaneously measuring the voltage and current flowing through the component.

We encounter another set of units in the study of magnetic fields. The magnetic flux density has the unit 1 T. From the relation for the Lorentz force acting on a charged particle moving in a magnetic field, we have the relation $1\,\mathrm{T} = 1\,\mathrm{N{\cdot}s{\cdot}C^{-1}{\cdot}m^{-1}} = 1\,\mathrm{kg{\cdot}A^{-1}{\cdot}s^{-2}}$. The unit of the magnetic flux is the weber $1\,\mathrm{Wb} = 1\,\mathrm{T{\cdot}m^2} = 1\,\mathrm{kg{\cdot}m^2{\cdot}A^{-1}{\cdot}s^{-2}}$. The last named unit is the henry $1\,\mathrm{H} = 1\,\mathrm{V{\cdot}s{\cdot}A^{-1}} = 1\,\mathrm{kg{\cdot}m^2{\cdot}s^{-2}{\cdot}A^{-2}}$, which describes the inductance of a configuration of conductors, that is, the constant of proportionality between the induced voltage and the change in electric current. Inductance is the main property of a coil. One way to determine these quantities is to calculate them from the known geometry of the conductors and the value of the currents flowing through them. The magnitude of a magnetic field can be measured using a magnetometer. The first magnetometers were similar to compasses – measuring the orientation and period of oscillation of a small dipole magnet on a torsion bar. In an external magnetic field, the magnet tries to orient itself in the direction of the lines of force – a small deflection from the equilibrium position can be calculated as

$$\alpha = \frac{pH}{D}\,,$$

where p represents the magnetic dipole moment of the magnet, H denotes the component of the magnetic field strength in the direction of the magnet, and D is the torsion spring constant, which we can determine by measuring the period of the torsional oscillations. To determine the angle, we can use the deflection of a reflected laser beam from a mirror placed on the magnet. However, to determine p, the instrument must first be calibrated, typically using a circular coil with a known value of the flowing current. Another possibility is to measure the induced voltage on a known coil when approaching the source of the magnetic field "from infinity". We have already encountered another method using the Hall effect by measuring the voltage on a Hall effect sensor.

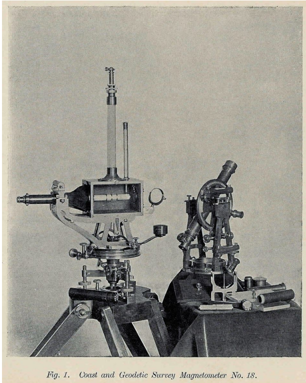

Figure 11: Land surveying magnetometer[15]

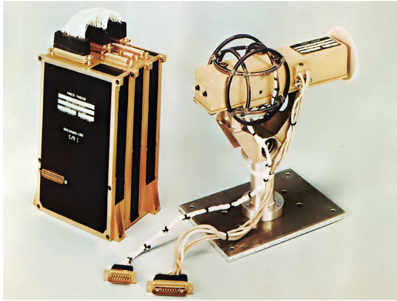

Figure 12: Magnetometer used in the Pioneer 10 and 11 probes[16]

A modern and highly sensitive measurement method is the so-called SQUID[17]. This device consists of a superconducting loop containing two oppositely oriented Josephson junctions. Due to its superconductivity, the value of the magnetic flux passing through this loop must be an integer multiple of the quantum Φ_0 of the magnetic flux

$$\Phi_0 = \frac{h}{2e}\,,$$

where h is Planck's constant and e is the elementary charge. The non-integer part of the fraction of flux generated by the external field Φ/Φ_0 is compensated by the generation of current by the circular loop. In addition, if sufficient external current flows through the loop, the voltage we measure on the loop is proportional to the value of the circulating current, which is a sinusoidal function of the magnetic field. This device, therefore, allows us to measure minuscule changes in the magnetic field down to $10^{-14}\,\mathrm{T}$, and with sufficient temporal resolution, to determine even higher values with this accuracy if we can determine the number of maxima of the sinusoid through which the voltage values have passed during the change. A noteworthy application of this method is magnetoencephalography – the measurement of brain activity by recording the magnetic fields at the level of $\mu\mathrm{T} - \mathrm{nT}$ accompanying the electrical currents present in the brain. In medicine, similar devices are also used to monitor heart or stomach activity.

After the preceding chapters, it should come as no surprise to the reader that the magnetic field can also be measured using optical methods. In this case, we owe it to the Zeeman effect, which causes the spectral lines of atoms to split. This effect enables us to measure magnetic fields at the surface of the Sun and stars.

[15] `https://commons.wikimedia.org/wiki/File:Coast_and_Geodetic_Survey_Magnetometer_Plate_XV_Fig_1_WBClark_1897.jpg`

[16] `https://en.wikipedia.org/wiki/File:Pioneer_10-11_-_P50_-_fx.jpg`

[17] Superconducting Quantum Interference Device

Other electromagnetic unit systems

As we have already mentioned, the emergence of electromagnetic units was not without its problems. The units were originally devised within the context of the CGS system, giving rise to two different types.

The electrostatic system CGS-ESU is based on Coulomb's law without any proportionality constant

$$F = \frac{q_1 q_2}{r^2}\,,$$

where for a charge of size 1 franklin (1 Fr) at a distance of 1 cm a force of 1 dyn is operating. The unit of current was thus $1\,\mathrm{Fr{\cdot}s^{-1}}$. The derived units in this system are usually called stat-ampere, stat-volt, and so on.

The CGS-EMU electromagnetic system is based on the action of conductors carrying current

$$\frac{F}{l} = \frac{I_1 I_2}{r}\,,$$

which leads to the definition of biot (abampere), which we mentioned earlier. In addition to the biot, in this system, we also encounter the gauss as a unit of magnetic flux density, the oersted as a unit of magnetic field strength, and the maxwell as a unit of magnetic flux. The transition from CGS to SI brought about the units used today by incorporating the factor $4\pi \cdot 10^{-7}\,\mathrm{H{\cdot}m^{-1}}$ into the permeability of vacuum, denoted as μ_0, which serves as the proportionality constant in the previous equation. However, with the new definition of the ampere, this value must be determined experimentally.

With the change in the units, the form of the equations also changed, especially the occurrence of permittivity and permeability and factors such as 2π and 4π. Even more confusion arises from the rationalization of some units, which, due to geometrical reasons, leave certain factors in equations. For instance, the Gaussian CGS units lack 4π in the Coulomb and Biot-Savart laws but include them in Maxwell's equations, the opposite of the SI system. The Heaviside-Lorentz system is similar to the SI system but takes both the values of permittivity and permeability of the vacuum equal to one. As a result, in some places, the speed of light appears explicitly in Maxwell's equations. One should, therefore, be extremely careful when using units other than from the SI system, even with the form of the relations used.[18]

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[18] The author, after personal experience, tries to avoid the CGS system altogether in non-mechanical contexts.